Practical Papers, Articles and Application Notes

Flavio Canavero, Technical Editor

The first article of this issue is an interesting contribution on the use of logarithmic units in the uncertainty evaluations of EMC measurements. It is authored by Dr. Carlo F.M. Carobbi of Florence University in Italy. In our community, the use of dBs is widespread to express most of the quantities (electrical and field levels, ratios, etc.) that are commonly employed in practice. Often, dBs are processed in a somewhat unorthodox manner, as if they were linear variables, and this brilliant contribution discusses the implications of the common practice of evaluating quantities expressed in logarithmic units; in particular, the statistics of dB-related uncertainties is discussed. I am looking forward to provoking an exciting discussion about this subject, and contributing to a fully-aware use of dBs inside the community.

The second article is entitled "Characterization of the Electromagnetic Environment in a Hospital: Measurement Procedures and Results" by a joint team of Swiss researchers from OFCOM and ETH-Zurich. This contribution describes an extensive survey of the electromagnetic environment from 9 kHz up to 10 GHz in the University Hospital in Zurich. One interesting result of this study is the determination of the link dynamic range of different wireless technologies which are potentially suited for short range applications such as patient monitoring networks.

In conclusion, I encourage (as always) all readers to actively participate to this column, either by submitting manuscripts they deem appropriate, or by nominating other authors having something exciting to share with the community. I will follow all suggestions, and with the help of independent reviewers I really hope to be able to provide a great variety of enjoyable and instructive papers. Please communicate with me, preferably by email at canavero@ieee.org.

The Use of Logarithmic Units in the Uncertainty Evaluations of EMC Measurements

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***Abstract*—The purpose of this contribution is to clarify some controversial aspects of the uncertainty evaluations when quantities expressed in logarithmic units are involved in the calculation. It is also shown that, when dealing with positive quantities having large relative uncertainties, it is convenient to pass to logarithmic units in order to restore the symmetry of the coverage interval around the best estimate. The fundamental role played by the log-normal distribution in the domain of linear units, as the counterpart of the normal one in the domain of logarithmic units, is described. The conceptual framework on which this contribution is based is that of the Guide to the Expression of Uncertainty in Measurement ([1], briefly known as the GUM), in that the law of propagation of uncertainty and the central limit theorem are assumed as the fundamental tools for uncertainty analysis.**

1. Introduction

The essential and distinctive features of the uncertainty evaluations of Electromagnetic Compatibility (EMC) measurements are: a) large uncertainties, and b) extensive use of logarithmic units.

Typical budgeted uncertainties for emission measurements (see [2] for an authoritative reference) amount to about 4 dB (conducted emissions) and to 5 dB (radiated emissions)[1]. These figures account for the uncertainty originated from the test method and the measuring instrumentation and neglect the variability due to the test set-up (mainly cables' layout) and the equipment under test. Including these last contributions uncertainties can easily double, reaching 10 dB or even more. Similar figures are expected to apply to the uncertainty of immunity test levels, such as in the case of radiated immunity or immunity to RF currents induced by bulk current injection.

The extensive use of logarithmic units in EMC (briefly log-units in the following) is explained by the wide dynamic range covered by both frequencies and amplitudes during EMC tests or, differently stated, by the need to instantaneously detect low and high quantity values on a broad band of frequencies. The log function expands the low values and compresses the high ones thus permitting their simultaneous and clear presentation on the same scale. The scale of measuring instruments, the calibration factor of measuring devices, their respective accuracy specifications, the test limits, the quantities appearing in the

[1] On the basis of two standard deviations.

mathematical equation describing the measurement model are all expressed in log-units. This obviously implies that the majority of the contributions appearing in an uncertainty budget and the final result of the uncertainty evaluation are expressed in log-units.

Although log-units are in use from the outset of EMC, their application to uncertainty evaluations is not perceived as straightforward by practitioners of test labs, and in some cases it is so, indeed. Some of the most debated subjects are:

- Correctness of the use of log-units in uncertainty budgets (e.g. sum of dB^2)
- Conversion of uncertainties from log-units to linear units and vice versa
- Mixing quantities expressed in log and in linear units
- Most appropriate unit for uncertainty: log or linear?
- How to deal with asymmetric uncertainty intervals

The scope here is to give guidance on the evaluation of EMC measurement uncertainty when log-units are involved. In doing this all the controversial issues listed above will be touched. Some of the material here presented is taken from or suggested by the conference papers [3] and [4].

2. Uncertainty Conversions from Log to Linear Units

Since the majority of EMC uncertainty evaluations are performed in log-units one may be interested in the conversion of the final result of the evaluation to linear units, for example in order to pass the information to people not acquainted with log-units. See Fig. 1 to this purpose. The horizontal scale is in linear units (X-axis), and the vertical one in log-units (Y-axis). The relation between X and Y is[2]

$$Y = 10 \log X \tag{1}$$

where X represents a power quantity (e.g. in mW), Y is the corresponding quantity expressed in absolute log-units (dBm) and 'log' is the base-10 logarithm. Note that an upper case letter, such as X and Y, is used to denote: a) the name of a quantity, b) the unique, although unknown, value of that quantity, c) any possible random value associated to that quantity. The meaning will be different depending on the context. A lower case letter is used for the best estimate of a quantity, hence x is the best estimate of X.

The best estimate of Y is $y = 6$ dBm and its expanded uncertainty is[3] $U(y) = 3$ dB. y is located at the center of the coverage interval (blue vertical segment along Y-axis), this is the typical occurrence since we assume that the uncertainty evaluation was directly performed in log-units. The best estimate of X is $x = 4$ mW and the bounds for the coverage interval (blue horizontal segment along X-axis) are 2 mW and 8 mW. The 6 dB = 9 dBm – 3 dBm range in log-units corresponds to the factor $4 = 8\ mW/2\ mW$ between the bounds in linear units. Note that y is the *arithmetic mean* of the log bounds while x is the *geometric mean* of the linear bounds. Also, the coverage interval in linear units is not symmetric, in that the best estimate is not halfway between the bounds. Finally, the log and the linear coverage intervals have the same coverage probability.

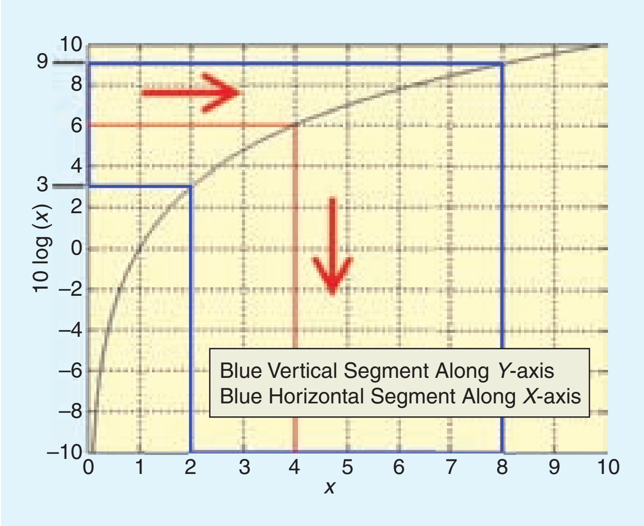

Fig. 1. Conversion from log to linear units.

We have therefore just derived some transformation rules: if y is the best estimate of Y, $U(y)$ is the uncertainty of y, that is Y is within $y - U(y)$ and $y + U(y)$ with a stated coverage probability, then $x = 10^{y/10}$ is the best estimate of X, $U_g(x)$ is the uncertainty of x and X is within $x/U_g(x)$ and $x \cdot U_g(x)$ with the same coverage probability. The lower script g stands for *geometric*.

Let us consider the type A uncertainty evaluation when performed in log-units. The best estimate is the arithmetic mean y of the N values $y_1, y_2, \ldots, y_N$,

$$y = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{2}$$

to which corresponds the geometric mean x_g in linear units

$$x_g = 10^{y/10} = \left(\prod_{i=1}^{N} x_i\right)^{1/N} \tag{3}$$

where $x_i = 10^{y_i/10}$. The uncertainty of the arithmetic mean is the *standard deviation of the mean*

$$u(y) = \sqrt{\frac{1}{N(N-1)}\sum_{i=1}^{N}(y_i - y)^2} \tag{4}$$

and the corresponding quantity in linear units is the *geometric standard deviation of the geometric mean*

$$u_g(x_g) = 10^{u(y)/10} \tag{5}$$

The intervals in Tab. 1 provide the same coverage probability. Also, if N is relatively large (say $N \geq 5$) y is approximately *normally distributed*, due to central limit theorem, then x_g is *log-normally distributed* and the coverage probabilities are nearly those in the last column in Tab. 1.

Now, let us come back to (4). The measurement unit of the deviation $y_i - y$ is dB, then $(y_i - y)^2$ is in dB^2 and $u(y)$ is again in dB. If we agree to accept (4) as the standard

[2] All the formulas appearing in the text are derived assuming the linear-to-log conversion (1). The necessary modifications for the case $Y = 20\log(X)$ are straightforward and left to the reader.

[3] The upper case letter U is used to indicate an expanded uncertainty while u denotes the standard uncertainty. Then $U = k\ u$, where $k > 1$.

TABLE 1. CONVERSION OF COVERAGE INTERVALS. THE COVERAGE PROBABILITY VALUES ARE OBTAINED ASSUMING THAT Y IS NORMALLY DISTRIBUTED SO THAT X IS LOG-NORMALLY DISTRIBUTED.

Log-Units	Linear Units	Coverage Probability (%)
$y - u(y) < Y < y + u(y)$	$x_g/u_g(x_g) < X < x_g \cdot u_g(x_g)$	68.3
$y - 2u(y) < Y < y + 2u(y)$	$x_g/u_g^2(x_g) < X < x_g \cdot u_g^2(x_g)$	95.4
$y - 3u(y) < Y < y + 3u(y)$	$x_g/u_g^3(x_g) < X < x_g \cdot u_g^3(x_g)$	99.7

uncertainty of y we have also to accept the correctness of the summation of quantities in dB^2 unit. To be more explicit, dB^2 may sound odd but the use of log-units in uncertainty evaluations is absolutely correct.

3. The Log-Normal Distribution

In EMC we have frequently to deal with the magnitude of a field, voltage or current or with a power, i.e. with intrinsically positive quantities. Let us consider a positive quantity X in linear units which is affected by a *large relative uncertainty*, that is $u(x)/x$ is not much less or even greater than unity. Then, if δ is a relatively large deviation, the probability that X may be less than $x - \delta$ or greater than $x + \delta$ is not small. Also, since X is positive, the probability that $X < x - \delta$ is less than the probability that $X > x + \delta$. It follows that the probability distribution of the possible values of X around x must be *asymmetric*. The point is that the proper displacement from x, providing equal probabilities in the tails of the distribution, is more convincingly obtained through *division and multiplication* by the same factor rather than *subtraction and addition* by the same deviation. This leads to the conclusion, at least in the absence of any other cogent information about X, that the probability that $X < x/m$ is the same as that $X > x \cdot m$, where $m \geq 1$. Symmetry is thus restored but in terms of the corresponding quantity Y in log-units. To summarize, if one has a sample of largely dispersed data[4] $x_1, x_2, \ldots, x_N$ of a positive quantity X, the most effective way to evaluate the best estimate x of X is through the geometric mean, while the dispersion of the data is calculated through the *geometric standard deviation* $u_g(X)$

$$u_g(X) = 10^{u(Y)/10} \tag{6}$$

where

$$u(Y) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(y_i - y)^2} \tag{7}$$

The standard uncertainty of $x = x_g$ is

$$u_g(x_g) = 10^{[u(Y)/\sqrt{N}]/10} \tag{8}$$

If the probability distribution of Y is symmetric (not necessarily normal) then y is the center value of the distribution and $x_g = 10^{y/10}$ is the *median* of the distribution of X.

Once obtained the best estimate of X and its standard uncertainty from the data sample, one is usually interested in deriving the coverage interval. To this purpose it is needed to assign a probability distribution to x_g. In the absence of any evidence leading to a different option, the choice falls on the log-normal distribution. In Fig. 2 the log-normal distribution corresponding to the parameters $y = 0$ and $u(y) = 2$ dB (corresponding to $x_g = 1$, $u_g(x_g) = 1.6$) is sketched. The distribution is asymmetric, however the coverage interval is straightforwardly obtained from the median x_g (see the blue vertical line in Fig. 2), and the geometric standard deviation $u_g(x_g)$ (the red vertical lines represent the lower and upper bounds of the 68.3 % coverage interval). By construction the area below each tail is the same (nearly 16 %, in the case represented in Fig. 2). It can be shown, see Appendix A, that the log-normal distribution is approximated by a normal distribution when the relative uncertainty of X is small. This suggests to assign the log-normal distribution to any positive random quantity when the best estimate and its standard uncertainty are evaluated through a series of measured data, regardless the magnitude of the relative uncertainty.

4. Uncertainty Conversions from Linear to Log-Units

Some quantities are more naturally expressed in linear rather than log-units. This is the case for a distance or the magnitude of the impedance of an artificial mains network. The best estimates of these quantities and their uncertainties must be converted from linear to log-units since generally the majority of the contributions in uncertainty budgets are in log-units and the final result of the uncertainty calculation must be expressed in log-units for comparison with a limit.

The conversion may be performed making use of approximated formulas. If x is the best estimate of X and $u(x)$ is the standard uncertainty of x, obtained through a type B uncertainty evaluation, the corresponding quantities in log-units are y and $u(y)$, where

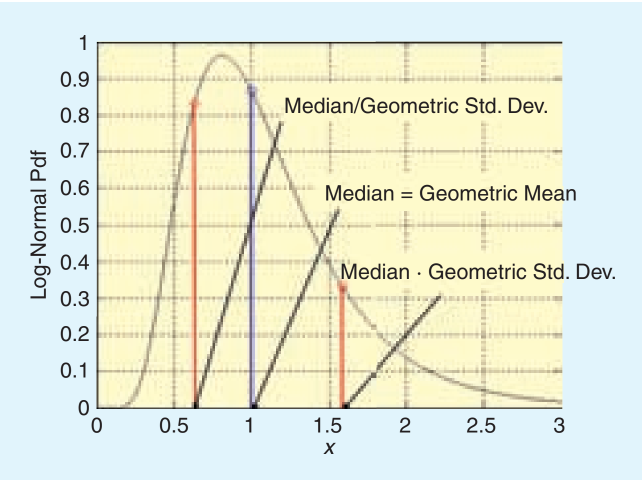

Fig. 2. Log-normal distribution.

[4] For example the 16 E-field values, in V/m, as measured with a broadband field meter over the uniform field area defined in the IEC 1000-4-3 immunity standard.

$$y = 10\log(x) - \frac{1}{2}\cdot 10\log(e)\cdot\left[\frac{u(x)}{x}\right]^2 \quad (9)$$

$$u(y) = 10\log(e)\cdot\frac{u(x)}{x} \quad (10)$$

and e is the Euler's number $(10\log(e) = 4.343)$. Note that, see (10), $u(y)$ is proportional to the relative standard uncertainty of x. Also, at the right-side of (9), a negative second order term appears which results from the fact that the log-function expands the lower values and compresses the higher ones. For example, if the best estimate of a correction factor is $x = 1$ and its relative standard deviation is $u(x)/x = 0.02$ (or 2 %), then from (9) $y = 0.00$ dB, and $u(y) = 0.09$ dB. If the best estimate is again $x = 1$ but its relative standard deviation is $u(x)/x = 0.2$ (or 20 %), then $y = -0.09$ dB, and $u(y) = 0.87$ dB.

Equations (9) and (10) provide an approximation of the best estimate and the standard deviation of the quantity in log-units, which is accurate when the relative uncertainty of the quantity in linear units is not exceedingly large (see the derivation of (9) and (10) in Appendix B). The inaccuracy of the prediction depends on the choice of the a-priori probability distribution of the quantity in linear units, with lower inaccuracy when the distribution is more concentrated about the best estimate. In

Appendix A

We show here that the log-normal distribution can be well approximated by the normal distribution when the relative standard uncertainty is small. Let us consider a random quantity X which follows the log-normal distribution with parameters η and σ^2. If we name $f_X(X)$ the distribution of X we have

$$f_X(X) = \frac{1}{\sqrt{2\pi}\sigma X}e^{-\frac{[\ln(X)-\eta]^2}{2\sigma^2}} \quad (A.1)$$

where we have assumed, for simplicity, the linear-to-log transformation $Y = \ln(X)$, where 'ln' is the natural logarithm. The parameters of the log-normal distribution can be linked to the expected value of X, $E\{X\}$, and the variance of X, $\mathrm{Var}\{X\}$ as follows [4, invert equations (4) and (5)]

$$\eta = \ln(E\{X\}) - \frac{1}{2}\ln\left(1 + \frac{\mathrm{Var}\{X\}}{E^2(X)}\right) \quad (A.2)$$

$$\sigma^2 = \ln\left(1 + \frac{\mathrm{Var}\{X\}}{E^2\{X\}}\right) \quad (A.3)$$

Since the relative standard uncertainty of X is small, we have $\sqrt{\mathrm{Var}\{X\}}/E\{X\} \ll 1$, then

$$\eta \approx \ln(E\{X\}) - \frac{1}{2}\frac{\mathrm{Var}\{X\}}{E^2(X)} \quad (A.4)$$

$$\sigma^2 \approx \frac{\mathrm{Var}\{X\}}{E^2\{X\}} \quad (A.5)$$

Substituting (A.4) and (A.5) into (A.1) we obtain, after manipulation,

$$f_X(X) \approx \frac{1}{\sqrt{2\pi\mathrm{Var}\{X\}}}\frac{E\{X\}}{X}e^{-\frac{\left[\ln\left(1+\frac{X-E\{X\}}{E\{X\}}\right)+\frac{1}{2}\frac{\mathrm{Var}\{X\}}{E^2\{X\}}\right]^2}{2\frac{\mathrm{Var}\{X\}}{E^2\{X\}}}} \quad (A.6)$$

The multiplying term $E\{X\}/X$ at the right-side of can be safely approximated by 1 and the logarithm in the argument of the exponential by $(X - E\{X\})/E\{X\}$. Further, taking into account that $1/2(\mathrm{Var}\{X\}/E^2\{X\}) \ll |X - E\{X\}|/E\{X\}$ and rearranging we have from (A.6)

$$f_X(X) \approx \frac{1}{\sqrt{2\pi\mathrm{Var}\{X\}}}e^{-\frac{(X-E\{X\})^2}{2\mathrm{Var}\{X\}}} \quad (A.7)$$

which is the normal distribution having expected value $E\{X\}$ and standard deviation $\sqrt{\mathrm{Var}\{X\}}$. Note that, when the relative dispersion is low, $E\{X\}$ is accurately estimated by both the arithmetic and geometric mean, and $\sqrt{\mathrm{Var}\{X\}}$ by the standard deviation and by $x_g[u_g(X) - 1]$, where x_g is the geometric mean and $u_g(X)$ is the geometric standard deviation as given by (3) and (6), respectively.

Appendix B

Equations (9) and (10) are here derived. The expectation and variance of a quantity $Y = g(X)$ are given by

$$E\{Y\} = \int_{+\infty}^{+\infty} g(X)f_X(X)dX \quad (B.1)$$

$$\mathrm{Var}\{Y\} = \int_{-\infty}^{+\infty} [g(X) - E\{Y\}]^2 f_X(X)dX \quad (B.2)$$

where $f_X(X)$ is the distribution of the random variable X. In [5, sec. 5–4] it is proved that if $g(X)$ is approximated by a parabola around $\eta = E\{X\}$, i.e.

$$g(X) \approx g(\eta) + g'(\eta)(X-\eta) + \frac{g''(\eta)}{2}(X-\eta)^2 \quad (B.3)$$

then

$$E\{Y\} \approx g(\eta) + g''(\eta)\frac{\sigma^2}{2} \quad (B.4)$$

$$\mathrm{Var}\{Y\} \approx |g'(\eta)|^2\sigma^2 \quad (B.5)$$

where $\sigma^2 = \mathrm{Var}\{X\}$. Now, substituting $g(X) = 10\log(X)$ into (B.4) and (B.5), equations (9) and (10) immediately follow (the correspondence between symbols is $\eta = x$, $\sigma^2 = u^2(x)$, $E\{Y\} = y$, $Var\{y\} = u^2(y)$)

the case where the quantity in linear units follows a rectangular distribution the inaccuracy, both on the best estimate and on the standard deviation, is less than 5 % if the half-width of the distribution is less than 30 %.

5. Conclusion

It should be borne in mind that the conversion from linear to log-units is a mathematical transformation, which is necessary in order to appropriately and simultaneously manage a large dynamic of values. Hence most of the controversy around the use of log-units in EMC uncertainty evaluation is probably due to lack of familiarity with mathematics and statistics. It is hoped that the information here conveyed will contribute, in part, to fill these gaps. The opinion of the Author is that the hard issues with uncertainty evaluations are not those related to computation (analytical or numerical), but the interpretation and use of the specifications provided by the manufacturers of measuring instruments and calibration laboratories and the (re) consideration of new or well-established measurement techniques in order to identify, quantify and possibly minimize the dominant contributions to uncertainty.

6. References

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Biography

Carlo F. M. Carobbi *(M'02) was born in Pistoia, Italy. He received the M.S. (cum laude) degree in electronic engineering and the Ph.D. degree in telematics from the University of Florence, Florence, Italy, in 1994 and 2000, respectively. Since 2001, he has been a Researcher in the Department of Electronics and Telecommunications, University of Florence, where he teaches the courses in electrical measurements and electromagnetic compatibility (EMC) measurements. His current research interests include EMC, and in particular EMC measurements and uncertainty evaluation, and EMC compliant design. Dr. Carobbi is a member of the IEEE Instrumentation and Measurement Society and the National Electrical and Electronic Measurements Association (GMEE), Italy.*

EMC