

Partial Inductance

Clayton R. Paul, Mercer University, Macon, GA (USA), paul_cr@Mercer.edu

Abstract—The increasingly important concept of partial inductance as opposed to loop inductance in high-speed, digital systems is discussed. Its use in explaining the concepts of “ground bounce” and “power rail collapse” in digital systems is given. Numerous other uses of partial inductance are given.

Index Terms—Inductance, current loops, partial inductance, magnetic flux, Faraday’s law

I. “Ground Bounce” and “Power Rail Collapse” in Digital Electronic Systems

We are entering a “digital age” where new concepts such as partial inductance are now needed to explain various “mysterious phenomena” and to adequately explain emerging logical dilemmas. For example, consider Fig. 1 which shows a CMOS inverter driving a capacitive load that perhaps represents the input to another CMOS inverter.

As the inverter switches from the LOW to the HIGH logic state, current I_{L-H} is momentarily drawn from the +5 V power supply through the upper PMOS, through the load capacitor and back to the power supply through the “ground” conductor thereby completing the complete current loop. Each of these lands connecting the circuit to the power supply is represented with inductors: L_{PR} and L_{GB} . Representing each land with its specific inductance as shown presents a logical dilemma that we will describe. As the currents through each inductor increases or decreases, voltages $L_{PR}(dI_{L-H}/dt)$ and $L_{GB}(dI_{L-H}/dt)$ are developed between the two ends of the land represented by that inductor. This causes the +5 V voltage at the power pin of the inverter to drop creating what is called “power rail collapse” which can cause logic errors. Similarly, the voltage of the “ground” pin of the module is caused to “bounce” which can also cause false logic errors. When the inverter switches low the capacitor current I_{H-L} discharges through the other ground land between the inverter and the capacitive load and the inductance of that land, L_{GB} , generating a voltage between its two ends that causes another “bounce”. In addition, if a cable shield is attached to the ground land (not recommended) the cable shield radiates radiated emissions like an antenna thereby causing EMC problems. We cannot eliminate these inductances; we can only mitigate their effect by placing decoupling capacitors between the +5 V and “ground” pins of the modules, and near to the module in order to minimize the land inductances connecting it to the module as shown in Fig. 2:

and/or placing the “going down” and return lands very close together (as partial inductances will show) or using ground and power “innerplanes”.

Figure 1 illustrates an important *logical dilemma* that the concept of partial inductances resolves. Throughout the technical literature, we see similar circuit diagrams wherein *sections of the interconnect conductors* are represented by discrete inductors BUT the *numerical values* of those discrete inductors are invariably never given or known by the authors. In fact these

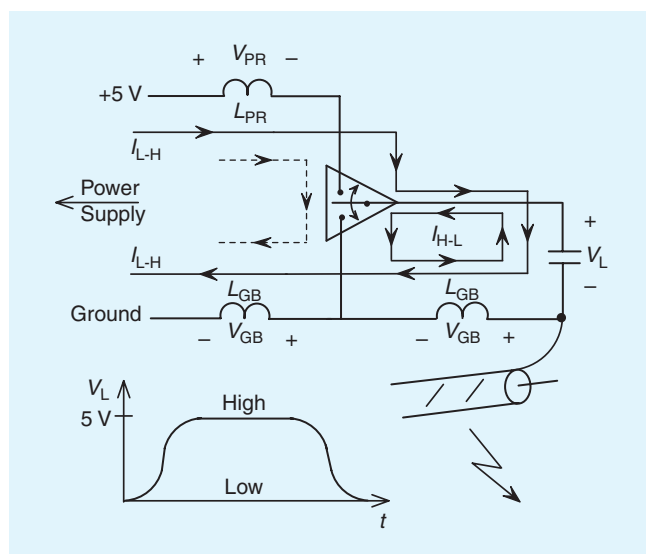


Fig. 1. Illustration of ground bounce and power rail collapse in a digital circuit.

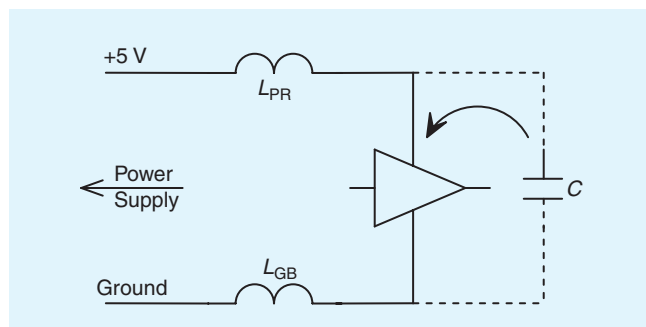


Fig. 2. Decoupling capacitors used to reduce the effect of land inductance.

discrete inductors are *partial inductances* which are the object of this paper.

In the first of a sequence of papers [1] the important concept of the inductance of a *closed loop* of a dc current was described.

Faraday's important law of induction provides that a *closed loop of current* has a *total inductance* of L_{loop} [1–5]. BUT *that total inductance of the loop cannot be uniquely placed anywhere in the loop!* So *how can we then uniquely assign portions of that total loop inductance to specific sections of the loop as shown in Fig. 1?*

If the loop is opened at any point, the total effect of that total loop inductance appears at the loop opening. If we inject a current into the input terminals of the loop, the current produces a magnetic flux density vector \vec{B} about the current having units of $\frac{\text{Webers}}{\text{m}^2}$ or Tesla. The direction of this magnetic field is determined by the *right-hand rule*. That is, if we place the thumb of our right hand in the direction of the current, the fingers will point in the direction of the magnetic field that forms concentric circles about the current. The total magnetic flux ψ having units of Webers due to this current that penetrates the open surface s enclosed by the closed contour c surrounding it is obtained with a *surface integral* as

$$\psi = \int_s \vec{B} \cdot d\vec{s} \quad \text{Webers} \quad (1)$$

A vector differential surface of that surface is $d\vec{s} = ds \vec{a}_n$ and \vec{a}_n is the unit normal to the surface. The *dot product* $\vec{B} \cdot d\vec{s}$ in the surface integral in (1) means that we take the product of the differential surfaces ds and the components of \vec{B} that are *perpendicular to the surface*. Then we add (with an integral) these products to give the *net magnetic flux ψ leaving (or passing through) the surface s* . This is a sensible result because \vec{B} has two components (as does any vector): one perpendicular to the surface and one that is tangent to the surface. The component of \vec{B} that is tangent to the surface does not contribute to the net flux passing through the surface. The *inductance of a current-carrying loop* is defined as *the ratio of the total magnetic flux penetrating the surface of the loop and the current of the loop that produced it*:

$$L_{\text{loop}} = \frac{\psi}{I} \quad \text{Henrys} \quad (2)$$

If we measure the voltage between two ends of a section of the loop conductor with a differential probe, we will see that the voltage drop will occur during the current pulse transition times (their rise/fall times) and not during the steady-state times of the current. If the voltage drop occurred during the steady-state times of the current it would be constant and would be due solely to the resistance of the land and not to the rate-of-change of the current. So the segment of the land in fact does possess an inductance of that segment. If we accept the fact that these voltages between two points on a conductor are due to the *derivative* of the current through it and not just *proportional* to the current, (as resistance would imply) and these voltages occur during the transitions of the logic pulses (during their rise/fall times) then we must accept the fact that we can *uniquely* attribute an inductance to a *segment of a current loop*. If we don't accept the fact that segments of a conductor have inductances that are uniquely attributable to them then we would have to tell circuit designers that "ground bounce and power rail collapse don't exist". Similarly we would have

no need for decoupling capacitors which no competent EMC engineer would remove from their designs!

In the second of the sequence of papers [2] we examined the concept of partial inductances of specific segments of that closed current loop. This allowed us to *uniquely allot* inductances L_{PR} and L_{GB} to specific segments of that closed loop and then to *uniquely* compute ground bounce and power rail collapse voltages across these inductances thereby resolving the dilemma of how to uniquely allot sections of the loop inductance to specific segments of that closed loop. In the next section we will summarize those important definitions and results and then apply them to solve important EMC problems that otherwise could not be solved or understood without the concept of partial inductance.

II. Definition of Partial Inductances

The definition and derivation of partial inductance is a bit tedious and relies on some vector concepts. The required vector calculus concepts are described in [2–4]. In order to *uniquely* define partial inductances of segments of a conductor requires that we define an auxiliary vector field known as the "vector magnetic potential" \vec{A} as

$$\vec{B} = \nabla \times \vec{A} \quad (3)$$

where $\nabla \times \vec{A}$ is the "curl" or circulation of \vec{A} as with "eddies" in a river. So if we can find \vec{A} we can obtain \vec{B} simply by vector manipulation and then obtain the total magnetic flux through the surface s and the inductance of the loop as in (1) and (2). Utilizing Stokes' theorem [2–4] we can alternatively determine the inductance of a closed loop, c , by integrating with a line integral the vector magnetic potential \vec{A} (produced by the current I) around the perimeter c of the loop instead of performing the surface integral of \vec{B} over the surface s of the loop surrounded by that contour as [2–4]:

$$L_{\text{loop}} = \frac{\psi}{I} = \frac{\int_s \vec{B} \cdot d\vec{s}}{I} = \oint_c \vec{A} \cdot d\vec{l} \quad (4)$$

The line integral adds the products of the differential path lengths around the contour, $d\vec{l}$, and the components of \vec{A} (produced by the current I) that are *tangent to the loop perimeter*. This is a sensible result since the vector magnetic potential has a component perpendicular to the loop perimeter and a component *parallel* to it (as does any vector). Only the component *parallel to it* contributes to the integral. It turns out that it is much easier to calculate the vector potential \vec{A} and then obtain \vec{B} simply by vector manipulation as in (3) than it is to directly obtain \vec{B} . The vector magnetic potential \vec{A} has two very important properties that greatly simplify its determination. These are:

- 1) The vector magnetic potential \vec{A} has a direction *parallel to the current I that produced it*, and
- 2) the vector magnetic potential \vec{A} goes to zero at infinity: $\lim_{r \rightarrow \infty} \vec{A} \rightarrow 0$.

The above development is the key to defining partial inductance of a segment of a current loop. Consider the rectangular loop shown in Fig. 3.

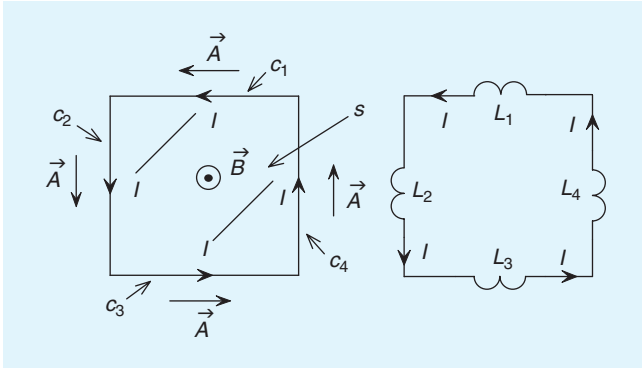


Fig. 3. Unique definition of partial inductance.

Utilizing (4) we can write the inductance of the loop as

$$\begin{aligned}
 L_{\text{loop}} &= \frac{\psi}{I} \\
 &= \frac{\int_s \vec{B} \cdot d\vec{s}}{I} \\
 &= \frac{\oint_c \vec{A} \cdot d\vec{l}}{I} \\
 &= \underbrace{\frac{\int_{c_1} \vec{A} \cdot d\vec{l}}{I}}_{L_1} + \underbrace{\frac{\int_{c_2} \vec{A} \cdot d\vec{l}}{I}}_{L_2} + \dots + \underbrace{\frac{\int_{c_n} \vec{A} \cdot d\vec{l}}{I}}_{L_n} \quad (5)
 \end{aligned}$$

Because the total magnetic flux through the loop defining the *total loop inductance* can be alternatively written as a line integral around the contour c enclosing the open surface s enclosed by the contour as in (5) we can *uniquely* break that integral into *unique* contributions attributable to specific sections of the loop perimeter thereby uniquely attributing specific contributions to the specific sections of the perimeter.

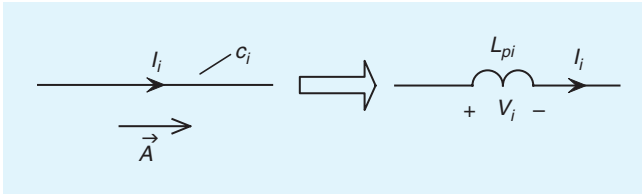


Fig. 4. The self partial inductance of a segment of a current loop.

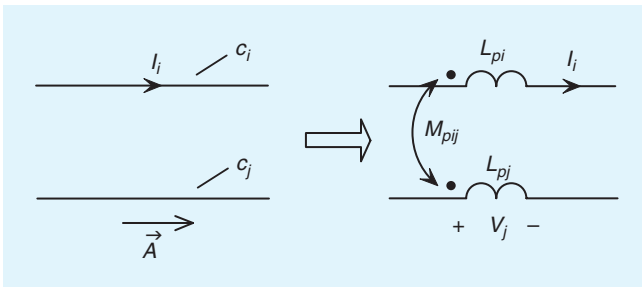


Fig. 5. The mutual partial inductance between two segments of a current loop.

With the above development we can uniquely define the self partial inductance of a segment of a closed current loop as shown in Fig. 4:

Hence the *self partial inductance* of a segment of a conductor is defined as

$$L_{pi} = \frac{\int_{c_i} \vec{A} \cdot d\vec{l}}{I_i} \quad (6)$$

and hence the voltage drop across that segment of the conductor is

$$V_i = L_{pi} \frac{dI_i}{dt} \quad (7)$$

The mutual partial inductance between two different segments of a current loop is similarly defined in Fig. 5.

Hence the *mutual partial inductance* between two different segments of a current loop is

$$M_{pij} = \frac{\int_{c_j} \vec{A} \cdot d\vec{l}}{I_i} \quad (8)$$

and the voltage drop across the second conductor segment is

$$V_j = M_{pij} \frac{dI_j}{dt} \quad (9)$$

III. The Physical Meaning of Partial Inductances

Although the previous section has given precise mathematical definitions of self and mutual partial inductances of segments of conductors, we will give the physical meaning of these partial inductances. Consider the current loop shown in Fig. 6. Draw two lines to infinity that are both perpendicular to the current. Then write around that loop

$$\begin{aligned}
 \frac{\psi_{\infty}}{I} &= \frac{\int_{s_i} \vec{B} \cdot d\vec{s}}{I} = \frac{\oint_c \vec{A} \cdot d\vec{l}}{I} \\
 &= \underbrace{\frac{\int_{c_i} \vec{A} \cdot d\vec{l}}{I}}_{L_{pi}} + \underbrace{\frac{\int_{\text{left side}} \vec{A} \cdot d\vec{l}}{I}}_{\text{left side}} + \underbrace{\frac{\int_{\text{right side}} \vec{A} \cdot d\vec{l}}{I}}_{\text{right side}} + \underbrace{\frac{\int_{\infty} \vec{A} \cdot d\vec{l}}{I}}_{\infty} \\
 &= \frac{\int_{c_i} \vec{A} \cdot d\vec{l}}{I} = L_{pi} \quad (10)
 \end{aligned}$$

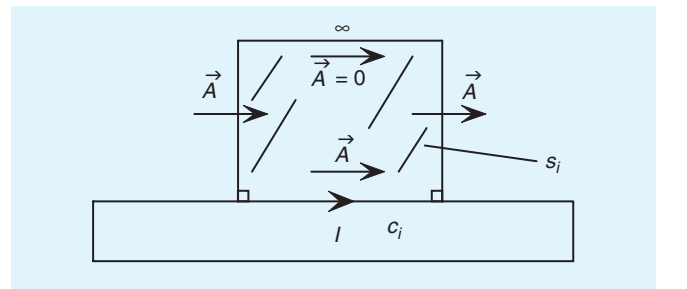


Fig. 6. The physical meaning of self partial inductance.

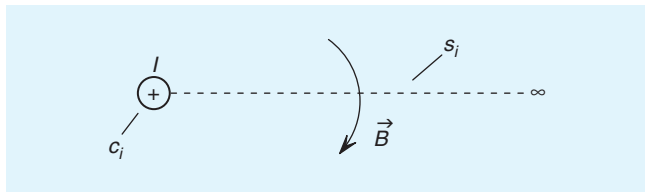


Fig. 7. The physical meaning of the self partial inductance.

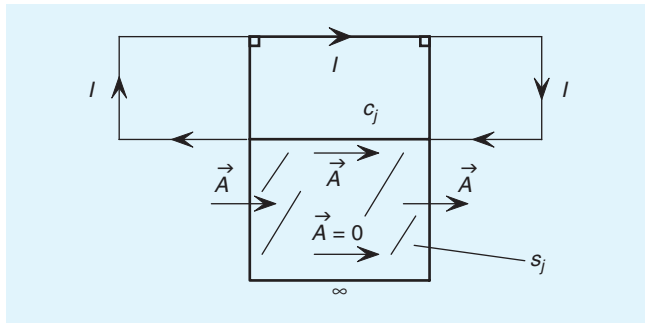


Fig. 8. The physical meaning of mutual partial inductance.

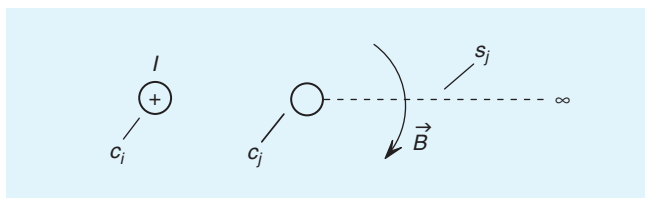


Fig. 9. The physical meaning of mutual partial inductance.

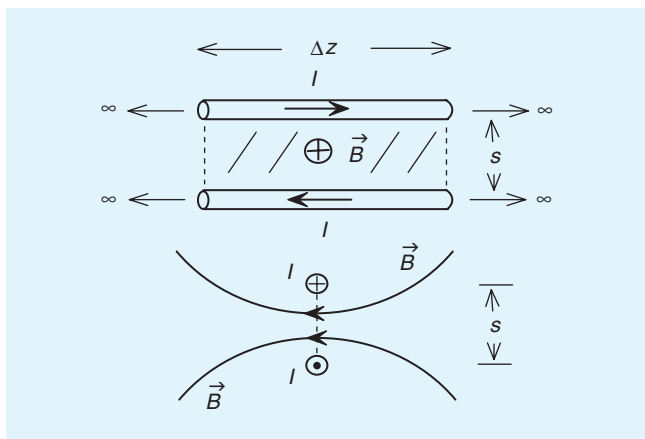


Fig. 10. A two-wire transmission line.

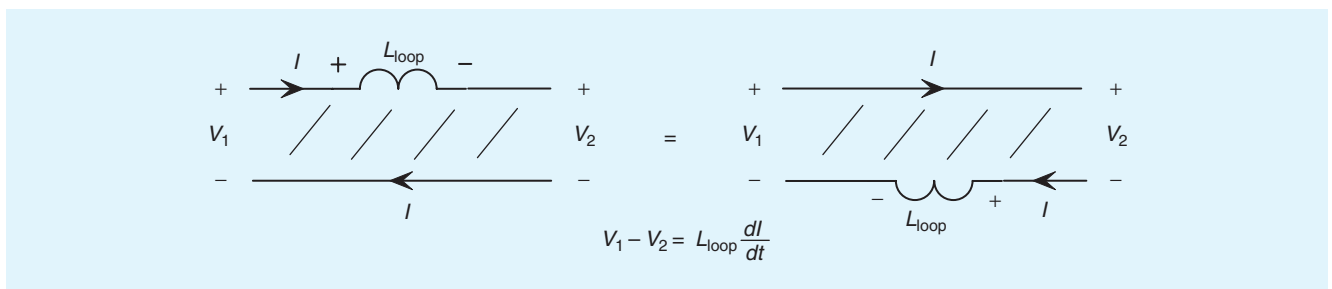


Fig. 11. Modeling a two-wire transmission line with its loop inductance.

The vector magnetic potential \vec{A} is perpendicular to the two sides (by construction) and so contribute nothing to the integral. Similarly the vector magnetic potential goes to zero at infinity and gives no contribution there. Hence we obtain the contribution only along the conductor segment.

Hence

The self partial inductance of a segment of the contour of a closed loop is the ratio of the magnetic flux penetrating the surface between that segment and infinity and the current on that segment as shown in Fig. 7.

The physical meaning of the mutual partial inductance is similarly obtained. Draw two lines to infinity that are perpendicular to the current on one segment and also enclose the area between the other segment and infinity as shown in Fig. 8.

Writing $\oint_c \vec{A} \cdot d\vec{l}$ around the loop between the second segment and infinity gives

$$M_{pij} = \frac{\int_{c_j} \vec{A}_{ij} \cdot d\vec{l}}{I} \quad (11)$$

The mutual partial inductance between two segments of one or more closed loops is the ratio of the magnetic flux (produced by the current on the first segment) that penetrates the surface between the second segment and infinity and the current on the first segment as shown in Fig. 9.

IV. Applications of Partial Inductance

A Transmission Line and its "Loop" Inductance

Consider the two-conductor transmission line of infinite length shown in Fig. 10.

The equal and oppositely-directed currents on the line produce magnetic fields that thread and contribute to the total magnetic flux through the loop area between the two conductors. That total "loop" inductance can then be placed in either conductor as shown in Fig. 11.

Placing the total "loop" inductance in either conductor gives the same terminal voltages. So how can we *uniquely* determine the voltage drop across either conductor such as the power rail collapse V_{PR} or ground bounce V_{GB} between the ends of either conductor?

However, if we model the two conductors with their individual partial inductances as shown in Fig. 12 we can determine the voltages between the ends of each conductor as

$$V_{PR} = L_{p1} \frac{dI}{dt} - M_p \frac{dI}{dt} = (L_{p1} - M_p) \frac{dI}{dt}$$

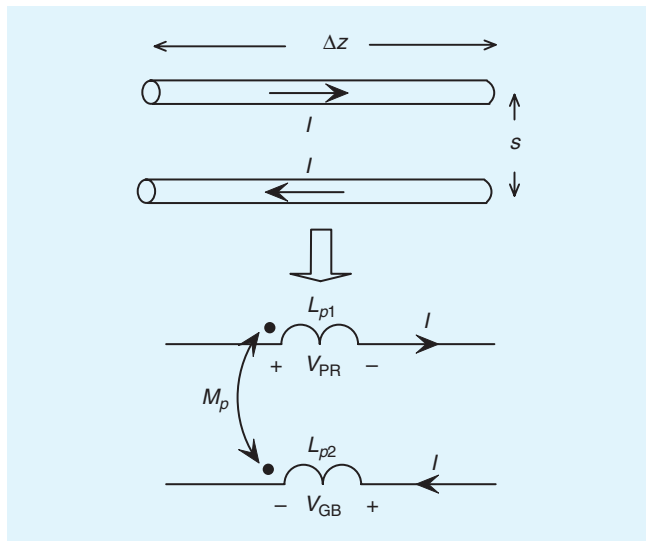


Fig. 12. Modeling each conductor with its unique partial inductance.

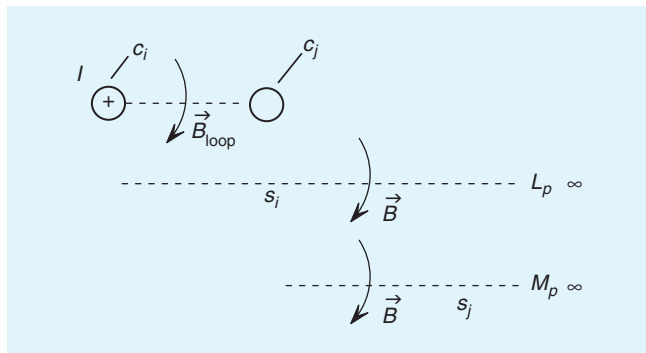


Fig. 13. Determining the loop inductance from the partial inductances.

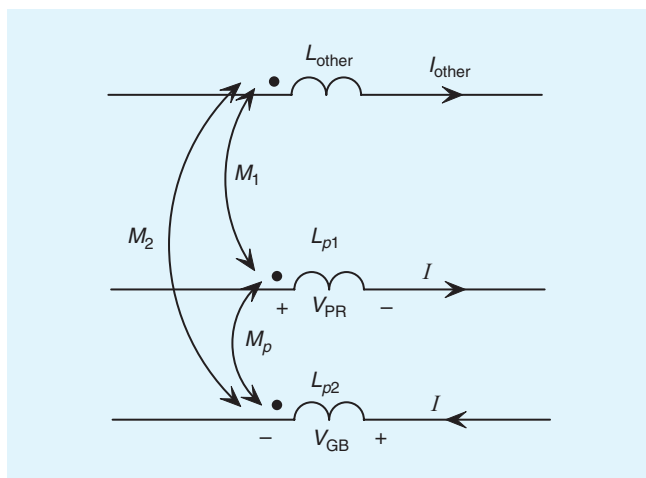


Fig. 14. Effects of neighboring currents.

$$V_{GB} = L_{p2} \frac{dI}{dt} - M_p \frac{dI}{dt} = (L_{p2} - M_p) \frac{dI}{dt}$$

As the two wires come closer $M_p \rightarrow L_{p1}$, L_{p2} and V_{GB} , $V_{PR} \rightarrow 0$. So place the power delivery and return conductors close together to reduce the power rail collapse and ground bounce voltages.

The loop inductance of a section of the line is

$$L_{loop} = L_{p1} + L_{p2} - 2M_p$$

$$L_{loop} = 2(L_p - M_p) \quad L_{p1} = L_{p2} = L_p$$

Interpreting the partial inductances as being the ratio of the magnetic flux between the segment and infinity and the current that produced it, the total magnetic flux that passes through the loop between the two transmission line conductors (the “loop” inductance) is proportional to the *difference* between the self and mutual partial conductors of the line conductors as shown in Fig. 13 so the result makes sense.

Effects of Neighboring Conductors

Nearby currents can affect these ground bounce and power rail collapse voltages as shown in Fig. 14.

Hence the total ground bounce and power rail voltages are influenced by these neighboring currents as

$$V_{GB} = L_{p2} \frac{dI}{dt} - M_p \frac{dI}{dt} - M_2 \frac{dI_{other}}{dt}$$

$$V_{PR} = L_{p1} \frac{dI}{dt} - M_p \frac{dI}{dt} + M_1 \frac{dI_{other}}{dt}$$

How can we compute V_{GB} and V_{PR} for this situation using “loop inductances”? What “loops” are we talking about and how can we determine their return paths on a complicated PCB?

Net Inductance of Wires in Parallel

It is commonly believed that placing two wires in parallel as in Fig. 15 will reduce the net inductance of the parallel combination to one-half that of one. This is NOT true. The *net* inductance of the combination is

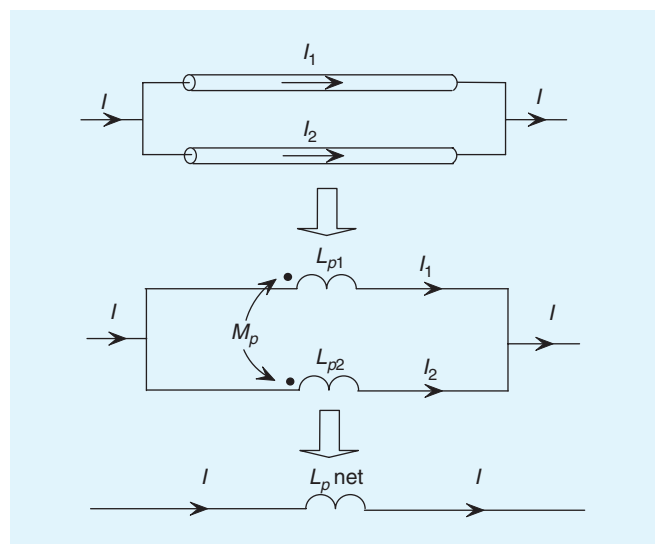


Fig. 15. Inductors in parallel.

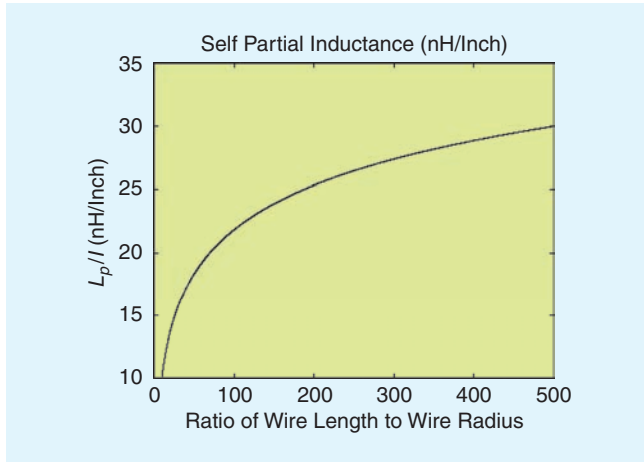


Fig. 16. Plot of L_p/l in nH/inch versus the ratio of wire length to wire radius l/r_w .

$$L_{p \text{ net}} = \frac{L_{p1}L_{p2} - M_p^2}{L_{p1} + L_{p2} - 2M_p}$$

$$L_{p \text{ net}} = \frac{L_p + M_p}{2} \quad L_{p1} = L_{p2} = L_p$$

As the separation increases, $M_p \rightarrow 0$ $L_{p \text{ net}} \rightarrow L_p/2$. So closely-spaced wires don't reduce their net inductance. In order to reduce the net inductance they must be widely spaced. This misconception occurs because we have forgotten about the mutual partial inductance between them.

Representative Values of the Partial Inductances

The self partial inductance of a wire of radius r_w and length l is [3]:

$$L_p = 2 \times 10^{-7} l \left[\ln \left(\frac{l}{r_w} + \sqrt{\left(\frac{l}{r_w} \right)^2 + 1} \right) - \sqrt{1 + \left(\frac{r_w}{l} \right)^2} + \frac{r_w}{l} \right]$$

$$\cong 2 \times 10^{-7} l \ln \left(\frac{2l}{r_w} - 1 \right) \quad (12a)$$

where we have assumed that the wire radius is much less than the wire length as is usually the practical case. Observe that this depends on the ratio of the wire length and the wire radius: l/r_w . Observe that the length of the wire, l , appears both inside and outside the equation. Hence it is not possible to speak of a per-unit-length inductance as is the case for a two wire transmission line of infinite length. Nevertheless we can divide both sides of (12) by the wire length and obtain a universal plot of the ratio of self partial inductance per unit length, L_p/l , versus the ratio l/r_w as

$$L_p/l = 5.08 \left[\ln \left(\frac{l}{r_w} + \sqrt{\left(\frac{l}{r_w} \right)^2 + 1} \right) - \sqrt{1 + \left(\frac{r_w}{l} \right)^2} + \frac{r_w}{l} \right] \frac{\text{nH}}{\text{inch}} \quad (12b)$$

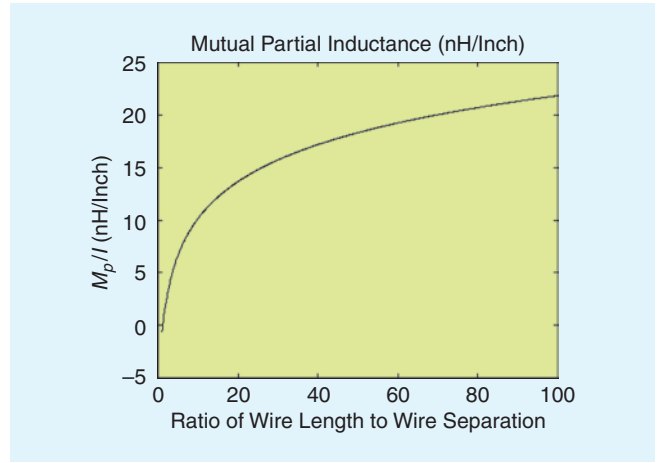


Fig. 17. Plot of M_p/l in nH/inch versus the ratio of wire length to wire separation l/s .

This is shown in Fig. 16 for ratios of $10 \leq l/r_w \leq 500$. For example, a #20 gauge (AWG) wire is a common wire size and has a radius of 16 mils. Hence the last plotted ratio of 500 represents a wire length of 8 inches for a #20 gauge wire (30.02 nH/inch), and a ratio of 10 represents a length of 0.16 inches or about $1/16$ of an inch (10.63 nH/inch). The plot in Fig. 16 indicates that a reasonable rule of thumb for practical wire sizes and wire lengths is a value of between 15 and 30 nH/inch.

The mutual partial inductance between two parallel wires of common length l and separation s is [3]:

$$M_p = 2 \times 10^{-7} l \left[\ln \left(\frac{l}{s} + \sqrt{\left(\frac{l}{s} \right)^2 + 1} \right) - \sqrt{1 + \left(\frac{s}{l} \right)^2} + \frac{s}{l} \right] \quad (13a)$$

Observe as was the case for self partial inductance, this depends on the ratio of wire length to wire separation, l/s . But the wire length, l , also appears outside the result so it is not possible to speak of a per-unit-length mutual inductance as is the case for a transmission line of infinite length. Nevertheless we can divide both sides of (13a) by the wire length and obtain a universal plot of the ratio of the per-unit-length mutual partial inductance, M_p/l , versus the ratio l/s as

$$M_p/l = 5.08 \left[\ln \left(\frac{l}{s} + \sqrt{\left(\frac{l}{s} \right)^2 + 1} \right) - \sqrt{1 + \left(\frac{s}{l} \right)^2} + \frac{s}{l} \right] \frac{\text{nH}}{\text{inch}} \quad (13b)$$

This is plotted in Fig. 17 for ratios of $1 \leq l/s \leq 100$. For example, a ratio of 80 would apply to two wires of length 5 inches and a separation between them of 0.0625 inches or $1/16$ of an inch (20.77 nH/inch), and a ratio of 10 would apply to two wires of length of 5 inches and a separation between them of $1/2$ of an inch (10.63 nH/inch). Observe that as the wire separation increases without bound, i.e., the ratio goes to zero, the mutual partial inductance goes to zero: an expected result. Similarly, as the wire separation goes to zero (approaches the radii of the wires), i.e., the ratio increases, the mutual partial inductance approaches the self partial inductance shown in Fig. 16: again, an expected result.

You Can Calculate the EXACT Inductance of Loops of Arbitrary Shapes

The loop inductance of the square loop shown in Fig. 18 can be obtained by adding the voltage drops around the loop:

$$V = 4(L_p - M_p) \frac{dI}{dt}$$

Substituting the equations for the self and mutual partial inductances in (12) and (13) gives:

$$\begin{aligned} L_{\text{loop}} &= 4(L_p - M_p) \\ &= 4 \frac{\mu_0 I}{2\pi} \left[\ln\left(\frac{2l}{r_w}\right) - 1 - \ln\left(\frac{l}{l} + \sqrt{1 + \frac{l^2}{l^2}}\right) + \sqrt{1 + \frac{l^2}{l^2}} - \frac{l}{l} \right] \\ &= \frac{2\mu_0 I}{\pi} \left[\ln\left(\frac{l}{r_w}\right) + \ln(2) - 1 - \ln(1 + \sqrt{2}) + \sqrt{2} - 1 \right] \\ &= \frac{2\mu_0 I}{\pi} \left[\ln\left(\frac{l}{r_w}\right) - 0.774 \right] \end{aligned} \quad (14)$$

which matches Grover's result [5]. This result can also be derived by calculating the total flux through the loop: The flux through the loop due to one of the current segments is the difference between the flux between that current and infinity, $L_p I$, and the flux between the opposite segment and infinity, $M_p I$, as determined previously. Hence the flux through the loop due to one of the segments is the difference $(L_p - M_p) I$. The total flux through the loop due to all 4 sides is $\psi = 4(L_p - M_p) I$.

The total magnetic field threading the loop can also be *very approximately* determined using the basic result for the B field for an *infinitely long wire* [3]:

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{infinitely long wire} \quad (15)$$

Approximate the loop perimeter as being constructed from four *infinitely long currents* and superimpose the four contributions to the total B fields of each given by (15). The magnetic flux

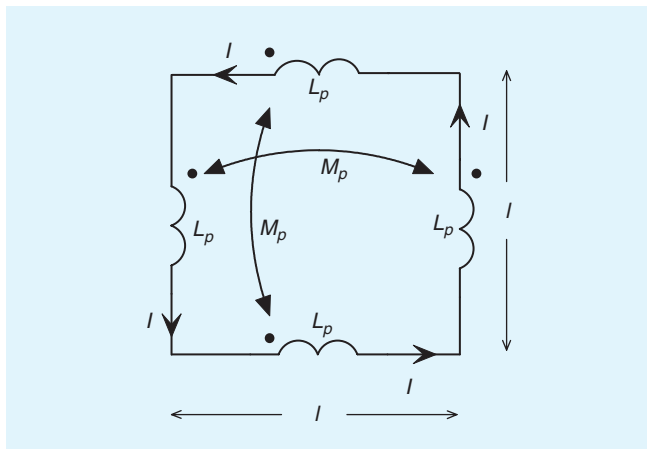


Fig. 18. Calculating inductance of arbitrarily-shaped loops with partial inductances.

through the loop due to the current of one side of length l is

$$\begin{aligned} \psi &= \int_s \vec{B} \cdot d\vec{s} \\ &= \int_{z=0}^l \int_{r=r_w}^l \frac{\mu_0 I}{2\pi r} dz dr \\ &= \frac{\mu_0 I l}{2\pi} \ln\left[\frac{l}{r_w}\right] \end{aligned}$$

Hence the total magnetic flux is four times this and the total loop inductance is approximately

$$L_{\text{loop}} \cong \frac{2\mu_0 I}{\pi} \ln\left(\frac{l}{r_w}\right)$$

Notice that the factor 0.774 in the *exact result* in (14) is missing here. This factor is the result of the fringing of the magnetic fields at the endpoints of the finite-length current element which the result for an infinitely-long current in (15) does not include. The B field for a *finite-length current element* does not maintain its circular behavior at its endpoints. The integration for determining the exact result in (14) including the fringing field at the endpoints is extremely complicated but has already been done in terms of partial inductances [3] and does not have to be repeated. That's the advantage of partial inductances.

Consider two segments of lengths l and m at an angle θ to each other and joined at a common point (or at least infinitesimally close) shown in Fig. 19. The mutual partial inductance between them is [3]

$$M_p = \frac{\mu_0}{4\pi} \cos\theta \left\{ l \ln\left[\frac{R + m - l \cos\theta}{l - l \cos\theta}\right] + m \ln\left[\frac{R + l - m \cos\theta}{m - m \cos\theta}\right] \right\} \quad (16a)$$

But this result can be put into an equivalent form as [3]

$$M_p = \frac{\mu_0}{4\pi} \cos\theta \left\{ l \ln\left[\frac{R + m + l}{R + l - m}\right] + m \ln\left[\frac{R + l + m}{R + m - l}\right] \right\} \quad (16b)$$

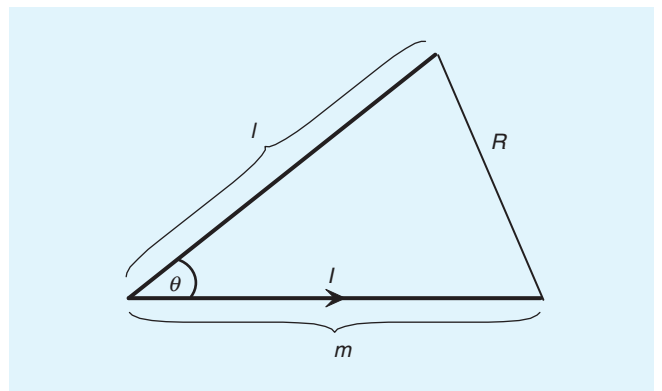


Fig. 19. Mutual partial inductance between two segments at an angle.

Next consider the equilateral triangle loop shown in Fig. 20. Writing the voltage V across one of the inductors gives

$$V = L_p \frac{dI}{dt} - 2M_p \frac{dI}{dt} = (L_p - 2M_p) \frac{dI}{dt} \quad (17)$$

The total voltage around the loop is three times (17) accounting for the voltages of all three sides. Hence the net loop inductance is

$$L_{\text{loop}} = 3(L_p - 2M_p) \quad (18)$$

Substituting the self partial inductances of the three wires from (12a):

$$L_p \cong \frac{\mu_0}{2\pi} l \left[\ln\left(\frac{2l}{r_w}\right) - 1 \right] \quad l \gg r_w$$

and the mutual partial inductances between two inclined wires of equal length from (16):

$$\begin{aligned} M_p &= \frac{\mu_0}{2\pi} \cos(60^\circ) \left[l \ln \frac{l+2l}{l} \right] \\ &= \frac{\mu_0}{2\pi} l (0.549) \end{aligned}$$

gives

$$\begin{aligned} L_{\text{loop}} &= 3(L_p - 2M_p) \\ &= 3 \frac{\mu_0}{2\pi} l \left[\ln\left(\frac{2l}{r_w}\right) - 1 - 2 \times 0.549 \right] \\ &= 3 \frac{\mu_0}{2\pi} l \left[\ln\left(\frac{l}{r_w}\right) + \ln(2) - 1 - 2 \times 0.549 \right] \\ &= 3 \frac{\mu_0}{2\pi} l \left[\ln\left(\frac{l}{r_w}\right) - 1.405 \right] \end{aligned} \quad (19)$$

which matches Grover's result [14].

Modeling Vias and Other Discontinuities on PCBs

Figure 21 shows a via barrel connecting an upper microstrip and a lower microstrip on two sides of a PCB connected by a via.

For a board of thickness of 64 mils and a barrel of radius 6.3 mils (#28 gauge) for a ratio of $l/r_w = 10.2$ gives a partial inductance of

$$\begin{aligned} L_p &= 5.08 l \left[\ln\left(\frac{l}{r_w}\right) + \sqrt{\left(\frac{l}{r_w}\right)^2 + 1} - \sqrt{1 + \left(\frac{r_w}{l}\right)^2} + \frac{r_w}{l} \right] \text{ nH} \\ &= 0.686 \text{ nH} \end{aligned}$$

giving an impedance of around 4.3Ω at 1 GHz.

V. Summary

THERE ARE A NUMBER OF PROBLEMS WITH USING "LOOP INDUCTANCE" THAT "PARTIAL INDUCTANCE" CIRCUMVENTS:

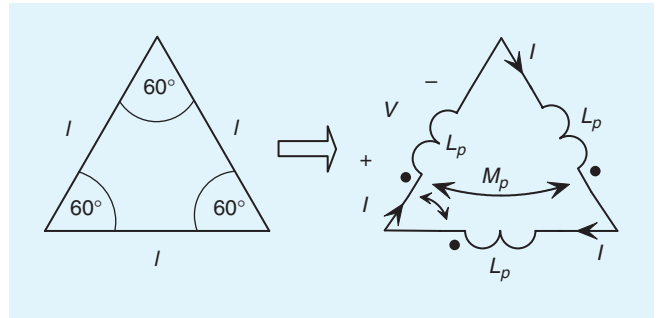


Fig. 20. The equilateral triangle loop.

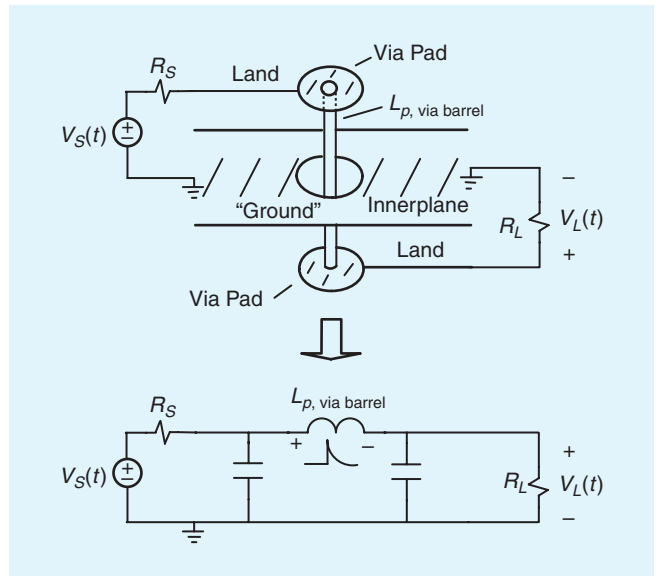


Fig. 21. Modeling a via with partial inductances.

- 1) To compute loop inductance, you MUST determine the RETURN PATH for the loop current. That's virtually impossible for most PCBs.
- 2) There is NO unique return path for ALL frequencies. At lower frequencies a current will return to its source along one path, while at higher frequencies that very same current will return along another path.
- 3) Even IF you could determine a unique loop for the current, where would you place the loop inductance in that loop? **There is NO UNIQUE answer!**
- 4) You cannot compute ground bounce and power rail collapse UNIQUELY with loop inductance.
- 5) Partial inductance solves ALL these problems. You just compute self and mutual partial inductances for all conductor segments of concern, place the self and mutual partial inductances in those conductor segments, and "turn the crank" by simply analyzing the resulting circuit. With this equivalent circuit you don't need to "guess" where the return for the current goes; you determine it from the equivalent circuit.

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Biography



Clayton R. Paul received the B.S. degree, from The Citadel, Charleston, SC, in 1963, the M.S. degree, from Georgia Institute of Technology, Atlanta, GA, in 1964, and the Ph.D. degree, from Purdue University, Lafayette, IN, in 1970, all in Electrical Engineering. He is an Emeritus Professor of Electrical Engineering at the University of Kentucky where he was a

member of the faculty in the Department of Electrical Engineering for 27 years retiring in 1998. Since 1998 he has been the Sam Nunn Eminent Professor of Aerospace Systems Engineering and a Professor of Electrical and Computer Engineering in the Department of Electrical and Computer Engineering at Mercer University in Macon, GA. He has published numerous papers on the results of his research in the Electromagnetic Compatibility (EMC) of electronic systems and given numerous invited presentations. He has also published 17 textbooks and Chapters in 4 handbooks. Dr. Paul is a Life Fellow of the Institute of Electrical and Electronics Engineers (IEEE) and is an Honorary Life Member of the IEEE EMC Society. He was awarded the IEEE Electromagnetics Award in 2005 and the IEEE Undergraduate Teaching Award in 2007.