The GUM Supplement 1 and the Uncertainty Evaluations of EMC Measurements

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Abstract—**The "Guide to the expression of uncertainty in measurement" (GUM) provides the theoretical framework which is worldwide adopted for the evaluation of measurement uncertainty, including the application to calibration and testing. The GUM uncertainty approach has however fundamental limits. If these limits are exceeded the results produced are no longer valid. The Supplement 1 to the GUM (GUMS1) describes a numerical technique aimed at extending the validity of the uncertainty evaluations to cases where the application of the GUM does not produce reliable results. The scope here is to point out the conditions under which it is necessary to make use of the GUMS1 technique. Explicit reference to Electromagnetic Compatibility (EMC) measurements will be made.**

1. Introduction

In 2008 Supplement 1 to the "Guide to the expression of uncertainty in measurements" (GUM) [1] was published by the Joint Committee for Guides in Metrology (JCGM). To the JCGM, which is chaired by the Bureau International des Poids et Mesures (BIPM), contribute the International Electrotechnical Commission (IEC), the International Federation of Clinical Chemistry and Laboratory Medicine (IFCC), the International Laboratory Accreditation Cooperation (ILAC), the International Organization for Standardization (ISO), the International Union of Pure and Applied Chemistry (IUPAC), the International Union of Pure and Applied Physics (IUPAP) and the International Organization of Legal Metrology (OIML). The Supplement 1 to the GUM (GUMS1) was prepared the Working Group 1 of the JCGM, which has the task to promote the use of the GUM and to prepare Supplements in order to broaden the range of its applications.

The GUMS1 provides an easy to implement numerical approach to the evaluation of measurement uncertainty named "propagation of distributions". The GUMS1 approach goes beyond the limits inherent to the GUM theoretical framework, which is based on the law of propagation of uncertainty and on the central limit theorem.

The GUMS1 is consistent with the GUM, in that the basic concepts and the terminology inspiring the guides are nearly the same. However some subtle, but deep, differences exist, the main consequence being the absence, in the GUMS1, of the Welch-Satterthwaite formula and of the concept of effective degrees of freedom [2, Appendix G].

The scope here is to clearly identify the situations where the application of the GUM uncertainty approach does not produce reliable results, and the numerical technique described by the GUMS1 is therefore needed. The operation of the propagation of distributions technique is briefly outlined and the few conceptual modifications introduced by the GUMS1 are also analyzed. An exemplification is offered through an application to Electromagnetic Compatibility (EMC) tests, in order to show how an uncertainty budget can be appropriately handled in the GUMS1 framework.

2. Uncertainty Evaluations in the GUM Theoretical Framework

The theoretical framework of the GUM is based on: a) the law of propagation of uncertainties (LPU), and b) the central limit theorem (CLT). In order that an uncertainty evaluation made according to the procedure described by the GUM may be correct the assumptions required for the validity of both LPU and CLT must be satisfied. Let us recall these assumptions starting from LPU.

2.1 The law of Propagation of Uncertainty

Let

$$Y = f(X_1, X_2, \ldots, X_N) \tag{1}$$

be the mathematical model linking the input quantities $X_1, X_2, \ldots, X_N$ to the output quantity Y. Y is the quantity of interest, for which we need to evaluate the best estimate y,

TABLE 1: LIST OF MONOMIAL EXPRESSIONS FREQUENTLY USED IN EMC. THE MEANING OF EACH SYMBOL IS THE FOLLOWING: E - DISTURBANCE ELECTRIC FIELD, AF - ANTENNA FACTOR, l_c- CABLE ATTENUATION, v_r- RECEIVED VOLTAGE, V- DISTURBANCE VOLTAGE, l_{amn}- ARTIFICIAL MAINS NETWORK VOLTAGE DIVISION FACTOR, P- DISTURBANCE POWER, l_{ac}- INSERTION LOSS OF THE ABSORBING CLAMP, R- INPUT RESISTANCE OF THE RECEIVER (50 Ω), e_t- TRANSMITTED ELECTRIC FIELD, P_t- TRANSMITTED POWER, G - GAIN OF THE TRANSMITTING ANTENNA, D - DISTANCE FROM THE EQUIPMENT UNDER TEST, Z - RESISTANCE OF EMPTY SPACE (377 Ω).

Linear units	Log-units
$e = v_r \cdot af \cdot l_c$	$E = V_r + AF + L_c$
$v = v_r \cdot l_{amn} \cdot l_c$	$V = V_r + L_{amn} + L_c$
$p = \frac{v_r^2}{R} \cdot l_{ac}^2 \cdot l_c^2$	$P = V_r + L_{ac} + L_c - 10log(R)$
$e_t = \frac{l_c}{d}\sqrt{\frac{Z}{4\pi} p_t \cdot g}$	$E_t = P + G_t + L_c - D + 10log(z/4\pi)$

its standard uncertainty $u(y)$, and a coverage interval having half-width $U(y)$. Let the inaccuracy of the mathematical model be smaller than the uncertainty we are going to estimate, and the best estimates $x_1, x_2, ..., x_N$ of each input quantity, and the corresponding standard uncertainties $u(x_1), u(x_2), ..., u(x_N)$, be available. Correlation between two ore more input quantities is neglected here for several reasons: a) correlation is unessential for the analysis developed, b) in order to keep the treatment as simple as possible and c) correlation is a side issue in EMC testing while it may be not always negligible in calibration[1], since uncertainties can be one order of magnitude lower than in testing.

Note that an upper case letter, such as X and Y, is used to denote: a) the name of a quantity, b) the unique, although unknown, value of that quantity, c) any possible random value associated to that quantity. The meaning will be different depending on the context.

LPU permits to obtain an estimate of $u(y)$ given by

$$u(y)=\sqrt{\left(\frac{\delta f}{\delta X_1}u(x_1)\right)^2 + \left(\frac{\delta f}{\delta X_2}u(x_2)\right)^2 + ... + \left(\frac{\delta f}{\delta X_N}u(x_N)\right)^2} \tag{2}$$

The partial derivatives are named *sensitivity coefficients* and they are evaluated at the best estimates $x_1, x_2, ..., x_N$. It is apparent from that the sensitivity coefficients are the *weights* of each standard uncertainty $u(x_i)$, contributing to the *combined* standard uncertainty $u(y)$. LPU provides an accurate estimate of $u(y)$ if the *non-linearity* of the function f is negligible in a neighborhood of each value $x_1, x_2, ..., x_N$ having half amplitude $u(x_1), u(x_2), ..., u(x_N)$, respectively. In such neighborhoods it must be verified that

$$f(X_1, X_2, ..., X_N) \approx f(x_1, x_2, ..., x_N) + \sum_{i=1}^{N} \frac{\delta f}{\delta X_i}(X_i - x_i) \tag{3}$$

and we say that the model is *quasi-linear*. In most cases the validity of (3) can only be demonstrated through numerical computation, with the exception represented by a very simple function f having few (say three at most) input quantities. A necessary, but not sufficient, condition for assuming negligible non-linearity is that

$$\frac{\delta f}{\delta X_i} \approx \frac{f(x_1,..., x_i + u(x_i),..., x_N) - f(x_1,..., x_i - u(x_i),..., x_N)}{2u(x_i)} \tag{4}$$

for any $i = 1, 2, ..., N$, which is less elaborate than (3). Equation (2) can be easily modified in order to take correlation into account (see [2, sec. 5.2]).

Oddly enough, due to the extensive use of log-units, model non-linearity is not frequent in EMC. Indeed in EMC, as in many other fields of engineering, the mathematical models describing the phenomena are in most cases monomial expressions, that is product of powers of quantities expressed in linear units. When converted to log-units a monomial is transformed into a linear combination of the same quantities expressed in log-units. In Tab. 1, left column, is reported a short list of monomial expressions in use in EMC for evaluation or prediction purposes. The corresponding expressions in log-units are in the right column.

Note that (see the log-units column in Tab. 1) the weights of the linear combination (i.e. the sensitivity coefficients) are all equal to ±1. This is due to the fact that the conversion to log-units takes into account the power or field nature of each quantity through the appropriate 10 or 20 multiplier. It is important to observe that one takes advantage of the use of log-units only when the model is a monomial expression. In the case of different types of non-linearity the only effect of the use of log-units is to further complicate the model. Examples of this kind of non-linearity are the uncertainty due to the imperfect knowledge of the distance between the equipment under test and the receiving antenna when reactive fields are not negligible and/or ground plane is present [3], the mismatch uncertainty [4] when both the VSWRs of the source and the receiver are large, the uncertainty due to the imperfect realization of the artificial mains network impedance (amplitude and phase) [5, sec. 6.4].

[1] Also of EMC measuring instruments and auxiliary equipments, devices and networks.

2.2 The Central Limit Theorem

CLT applies when: a) the model is linear or quasi-linear, that is it must be verified, at least approximately, that

$$Y = c_0 + c_1X_1 + c_2X_2 + ... + c_NX_N \quad (5)$$

b) input quantities are independent, c) $|c_iu(x_i)|$ have comparable magnitude, d) N is sufficiently large (say $N \geq 3$). If the requirements a) through d) are satisfied then Y approximately follows a normal distribution having expected value y and standard uncertainty $u(y)$, where

$$y = c_0 + c_1x_1 + c_2x_2 + ... + c_Nx_N \quad (6)$$

and

$$u(y) = \sqrt{[c_1u(x_1)]^2 + [c_2u(x_2)]^2 + ... + [c_Nu(x_N)]^2} \quad (7)$$

Note that the combined uncertainty $u(y)$ is the root-sum-of-the-square of the weighted contributing uncertainties[2]. Due to the nature of the quadratic summation it may easily happen that one contribution is dominant, especially if N is small (say 2, 3, 4). In this case the probability distribution of y is nearly that of the dominant contribution. If this last is non-normal then y is non-normal too and CLT does not apply.

To summarize we have that:

A. If the model is not linear or quasi-linear LPU does not apply and CLT does not apply

B. If the model is linear or quasi-linear and few non-normal contributions are dominant LPU applies and CLT does not apply

Note that in the case B we can obtain an accurate estimate of $u(y)$ but the calculation of the coverage interval is not immediate since we do not know the probability distribution of Y. In this case we have to make use of very general[3] inequalities like the Tchebycheff inequality or the Chernoff bound [6, pp. 113–115] which provide looser bounds (wider coverage intervals), for a stated coverage probability, than those obtained from the normal distribution. A pragmatic approach is that followed in [7, Appendix C] where tables are given providing coverage coefficients[4] for all the combinations in pairs[5] of rectangular, normal and U-shaped probability distributions, for various values of the ratio of their corresponding standard deviations. It is a method which lacks of generality and it is rather inelegant, but effective when the available options apply. In the case A we cannot even obtain a reliable estimate of $u(y)$. GUMS1 provides a solution both in the case A and in the case B.

3. Uncertainty Evaluations by Using the GUMS1 Numerical Approach

The approach to the evaluation of measurement uncertainty described by the GUMS1 essentially consists in the numerical implementation of the Monte Carlo method. The technique of the *propagation of distributions*, a rather evocative terminology to distinguish from the propagation of uncertainties, permits to numerically obtain the probability distribution of Y propagating the probability distributions of the input quantities through the model (1). The technique works both in the case of uncorrelated or correlated input quantities, linear or non-linear model, presence or absence of one (few) dominant contribution(s).

A sample of length M of random numbers $X_{i,(1)}, X_{i,(2)}, \ldots, X_{i,(M)}$ is numerically generated for each input quantity X_i ($i = 1, 2, ..., N$) according to its probability distribution. If, for ease of notation, we define $\mathbf{X} = X_1, X_2, ..., X_N$, so that $\mathbf{X}_{(j)} = X_{1,(j)}, X_{2,(j)}, ..., X_{N,(j)}$, then we have

$$Y_{(j)} = f(\mathbf{X}_{(j)}) \quad (8)$$

for $j = 1, 2, ..., M$. From the sample of output values $Y_{(1)}, Y_{(2)}, ..., Y_{(M)}$ a discrete approximation of the continuous probability distribution of Y is readily obtained. The value M is adaptively selected through a procedure aimed at obtaining a required tolerance on the coverage interval. Typically [1, note in 7.2.1], a value of M of the order of 10^6 is expected to deliver a 95% coverage interval for Y such that its length is correct to one or two significant decimal digits. Random number generators are common and reliable tools provided by the commercial software for numerical and statistical analysis. The time needed for processing the inputs in order to obtain the probability distribution of the output largely depends on the complexity of the model and on the sample length M, however typically this time does not exceed few seconds.

Apart from the details of the numerical approach proposed by the GUMS1, what is important here is to draw attention to some general considerations. Firstly, the GUMS1 technique gets over the limitations inherent to the GUM uncertainty approach: mainly the inability to appropriately deal with model non-linearity and with the presence of one or few dominant uncertainty contributions. Also correlation (once identified and quantified) can be easily added to the recipe. Secondly in the GUMS1, the GUM concept of degrees of freedom as a measure of the reliability of an uncertainty estimate[6] is abandoned. The odd practice of assigning an infinite number of degrees of freedom to type B uncertainty evaluations, the "original sin" of type A evaluations having, by necessity, a small number of degrees of freedom, and the Welch-Satterthwaite formula do not exist in the GUMS1.

Both in the case where: a) the (effective) number of degrees of freedom comes along with the best estimate of a quantity and its standard uncertainty from a calibration report, and b) a quantity is estimated through repeated measurements, a *shifted and scaled Student's t* probability distribution with ν degrees of freedom is assigned to that quantity. The shifting parameter is the best estimate, the scaling parameter is the standard uncertainty, and the number of degrees of freedom is the value in the calibration report or the number of measurements minus one. This assignment is a direct consequence of the application of the Bayes theorem to the statistical inference of the unknown value of a quantity starting from the available prior knowledge. Once adopted the Bayesian approach the uncertainties derived from the available information are exact, regardless of their type A or

[2] Just the same as in (2).

[3] In that they are applicable to any probability distribution or to wide categories of distributions.

[4] Corresponding to 95.45% coverage probability.

[5] Except the normal-normal combination, which is again normal.

[6] The uncertainty of uncertainty.

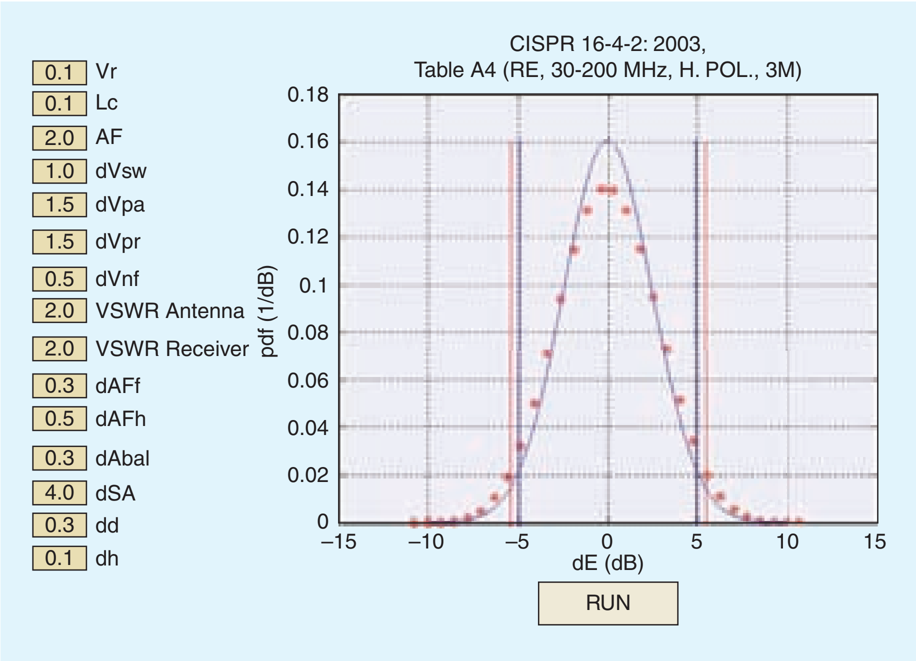

Fig. 1. GUI used to interact with the routine implementing the GUMS1 technique and the GUM approach to the evaluation of measurement uncertainty. The case represented here is that of radiated emission measurements. The uncertainty budget is that in [9, Table A.4]. The meaning of each contribution is the following: Vr - Receiver reading (normal, k = 1), Lc - Attenuation: antenna-receiver (normal, k = 2), AF - Biconical antenna factor (normal, k = 2), dVsw - Receiver correction for sine wave voltage (normal, k = 2), dVpa - Receiver correction for pulse amplitude response (rectangular), dVpr - Receiver correction for pulse repetition rate response (rectangular), dVnf - Receiver correction for noise floor proximity (normal, k = 2), VSWR Antenna - VSWR of antenna input (contribution to mismatch uncertainty), VSWR Receiver - VSWR at receiver input (contribution to mismatch uncertainty), dAFf - Biconical antenna correction for AF frequency interpolation (rectangular), dAFh - Biconical antenna correction for AF height deviations (rectangular), dAbal - Biconical antenna correction for balance (rectangular), dSA - Site correction for site imperfections (triangular), dd - Site correction for separation distance (rectangular), dh - Site correction for table height (normal, k = 2).

type B evaluation. It is likely that also the GUM will undergo a significant revision according to the concepts here outlined [8].

4. Application of the GUMS1 to EMC Measurement Uncertainty Evaluations

The scope here is to show how an uncertainty budget and the calculation of the coverage interval can be handled by using the GUMS1 technique. For sake of comparison the results obtained through the application of the GUM procedure are also reported. The case considered is that of radiated emission testing, in the 30–200 MHz frequency range, horizontal polarization, 3 m distance. Reference is made to [9, Table A.4].

In Fig. 1 is represented the Graphical User Interface (GUI) created by using Matlab® and designed to interact with the routine implementing both the GUMS1 technique and the GUM procedure. The values of the uncertainty contributions are those in the column in the left side of the figure. They are the same as those appearing in [9] and adopted there to calculate the reference uncertainty value U_{CISPR}. For the explanation of the meaning of each contribution and the corresponding probability distribution see the caption of Fig. 1. In the plot of the same figure is represented the normal probability distribution (blue continuous line), as it is obtained through the GUM approach, and the discrete probability distribution (red circles with dots) calculated with the GUMS1 technique. The vertical bars represent the 95.45 % coverage intervals corresponding to the two probability distributions (blue vertical bars for the GUM distribution, red ones for the GUMS1 distribution). The coverage interval delimited by the red bars is the probabilistically symmetric one[7]. It is evident from the plot that we obtain nearly the same results by using the GUM and the GUMS1 approaches. The smallest and the largest values in the coverage interval are −5.0 dB and +5.0 dB (GUM) or −5.4 dB and +5.5 dB (GUMS1) the difference being only 0.9 dB. It is assumed that the internal attenuation of the receiver is 0 dB and no matching pad is placed at the output terminals of the receiving antenna. In this case the VSWR at the receiver input may reach 2.0 to 1 (see [10, section 4.1]). The VSWR at the antenna terminals is 2.0 to 1, that is a relatively moderate value.

Let us now change the antenna VSWR to 30 to 1 (as in the case of the biconical antenna at few tens of megahertz, without any matching pad). All the other contributions to measurement uncertainty are left the same. The corresponding probability distribution is that plotted in Fig. 2. It is evident, see the red plot, that the mismatch uncertainty contribution is dominant, since the shape of the distribution is strongly affected by the resultant *asymmetric* U-shaped distribution [11]. The probability distribution obtained through the GUMS1 technique largely deviates from the normal distribution obtained following the GUM procedure. Also the coverage intervals are very different. Their extreme values are ±6.5 dB (GUM) and −7.9 dB, +8.9 dB (GUMS1). In conclusion the GUM approach provides a conservative estimate of the coverage interval, which deviates from the GUMS1 estimate by about 4 dB.

[7] The probabilistically symmetric coverage interval is such that the probability that Y is less than the smallest value in the interval is equal to the probability that Y is greater than the larger value in the interval [1, definition 3.15].

[8] Although the two options of the probabilistically symmetric and the shortest length coverage interval are presented in the GUMS1.

Conclusion

In the GUMS1 is described a numerical technique which permits to obtain the probability distribution of the quantity of interest also in those cases where the GUM uncertainty approach does not work, namely when the model non-linearity is not negligible and/or when one or few non-normal uncertainty contributions are dominant.

Two remarks are however worth noting here. The first is that if the probability distribution of the output quantity, obtained through the application of the GUMS1 technique, is asymmetric and/or multi-modal the choice of the best estimate and of the coverage interval is in any case left to the user of the method. The GUMS1 does not provide rules to follow in these circumstances[8] simply because they do not exist, the optimal solution depending on the specific problem at hand. The second is that it is not convenient to adopt the GUMS1 technique as the standard practice for uncertainty evaluation. This is because, being an entirely numerical technique, you lose insight into the measurement process, making more difficult the identification and analysis of the most significant contributions to the combined uncertainty.

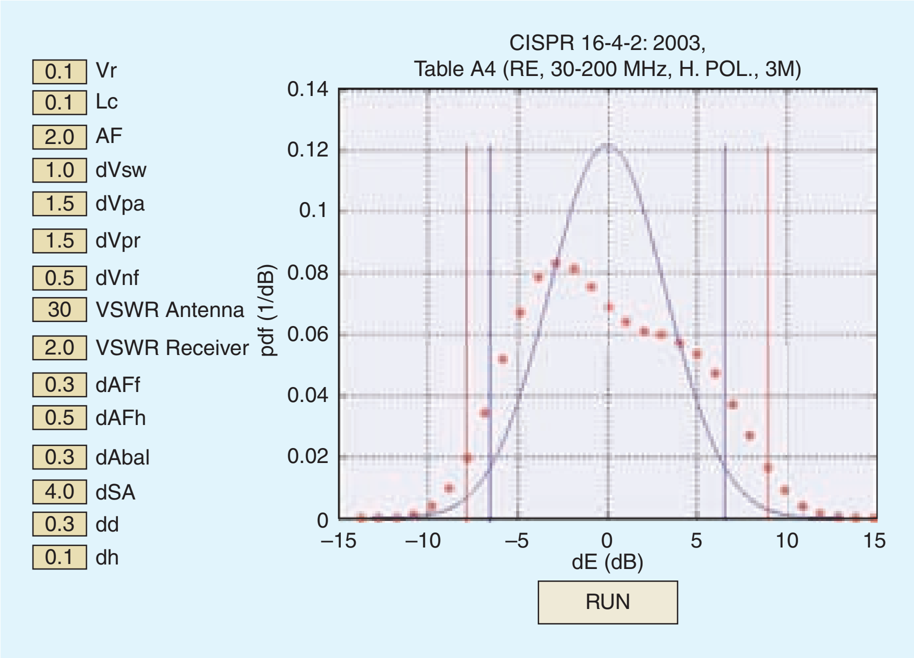

Fig. 2. The same as Fig. 1 but with the VSWR at the antenna terminals equal to 30 to 1 instead of 2 to 1.

References

[1] Joint Committee for Guides in Metrology (JCGM), "Evaluation of measurement data – Supplement 1 to the 'Guide to the expression of uncertainty in measurement' – Propagation of distributions using a Monte Carlo method", JCGM 101:2008, First edition 2008. Freely available at OIML web site, www.oiml.org.

[2] Joint Committee for Guides in Metrology (JCGM), "Evaluation of measurement data – Guide to the expression of uncertainty in measurement", JCGM 100:2008, First edition 2008. Freely available at OIML web site, www.oiml.org.

[3] Luigi M. Millanta, Carlo F.M. Carobbi, Marco Cati, "Statistical Treatment of Experimental Data", Int. Symp. EMC Europe 2004, Sep. 6-10, 2004. Invited contribution to the workshop "Statistical Approaches in EMC design, Computation and Testing", pp. 126–131.

[4] I. A. Harris and F. L. Warner, "Re-examination of mismatch uncertainty when measuring microwave power and attenuation", IEE Proc., Vol. 128, Pt. H, No.1 , pp. 35-41, Feb. 1981.

[5] International Electrotechnical Commission (IEC) – International Special Committee on Radio Interference (CISPR), "Specification for radio disturbance and immunity measuring apparatus and methods – Part 4-1: Uncertainties, statistics and limit modeling – Uncertainty in standardized EMC tests", CISPR/TR 16-4-1, Second edition, Feb. 2009.

[6] Athanasios Papoulis, "Probability, Random Variables, and Stochastic Processes", Third edition, McGraw-Hill series in electrical engineering, communications and signal processing, 1991. ISBN 0-07-048477-5.

[7] United Kingdom Accreditation Service (UKAS), "The Expression of Uncertainty and Confidence in Measurement", UKAS M3003, Edition 2, Jan. 2007.

[8] Walter Bich, "How to revise the GUM?", Accreditation and Quality Assurance, Vol. 13, Numbers 4-5, May 2008, pp. 271–275(5).

[9] International Electrotechnical Commission (IEC) – International Special Committee on Radio Interference (CISPR), "Specification for radio disturbance and immunity measuring apparatus and methods – Part 4-2: Uncertainties, statistics and limit modeling – Uncertainty in EMC measurements", CISPR 16-4-2, First edition, Nov. 2003.

[10] International Electrotechnical Commission (IEC) – International Special Committee on Radio Interference (CISPR), "Specification for radio disturbance and immunity measuring apparatus and methods Part 1-1: Radio disturbance and immunity measuring apparatus - Measuring apparatus", CISPR 16-1-1, Second edition, Mar. 2006.

[11] Darren Carpenter, "A Demystification of the U-Shaped Probability Distribution", in Proc. 2003 IEEE Int. Symp. on EMC, Boston, (MA), pp. 521-525, 18–22 Aug. 2003.

Biography

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