The Sixth Most Referenced Transactions Paper of the EMC Society

Dan Hoolihan, History Committee Chair

INTRODUCTION

As a continuing tribute to the 50th Anniversary Celebration of the EMC Society of the IEEE (1957-2007), we are republishing past top papers from the *IEEE Transactions on Electromagnetic Compatibility*. In the five previous issues of the EMC Newsletter, we have published the first five most referenced papers, which are respectively:

1. "Transient Response of Multiconductor Transmission Lines Excited by a Nonuniform Electromagnetic Field;" EMC-22, No. 2, May - 1980, Page 119 by A. K. Agrawal, H. J. Price, and S. H. Gurbaxani.
2. "Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic Field Equations;" EMC-23, No. 4, November - 1981, Page 377 by Gerrit Mur.
3. "Generation of Standard Electromagnetic Fields Using TEM Transmission Cells;" EMC-16, No. 4, November - 1974, Pages 189 -195 by Myron (Mike) L. Crawford.
4. "Frequency Response of Multiconductor Transmission Lines Illuminated by an Electromagnetic Field," EMC-18, No. 4, November - 1976, Pages 183-190 by Clayton R. Paul.
5. "Statistical Model for a Mode-Stirred Chamber," EMC-33, No. 4, November - 1991, Pages 366-370 by Joseph G. Kostas and Bill Boverie.

In this issue, we are publishing the sixth most-referenced *IEEE Transactions on EMC* paper of the first fifty years of the EMC Society. It is written by Henning Harmuth and was a controversial paper when written in the mid-1980s.

The title of the paper is "Correction of Maxwell's Equations for Signals I" and it was first published in the *IEEE Transactions on EMC* in Volume 28, No. 4, November 1986.

Again, we hope you take the time to read and appreciate the significance of this historical article.

Correction of Maxwell's Equations for Signals I

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Abstract—**Electromagnetic wave theory has been based on the concept of infinitely extended periodic sinusoidal waves ever since Maxwell published his theory a century ago. On the practical level this worked very well, but on the theoretical level we always had an indication that something was amiss. There was never a satisfactory concept of propagation velocity of signals within the framework of Maxwell's theory. The often-mentioned group velocity fails on two accounts, one being that it is almost always larger than the velocity of light in radio transmission through the atmosphere; the other being that its derivation implies a transmission rate of information equal to zero. A closer study shows that Maxwell's equations fail for waves with nonnegligible relative frequency bandwidth propagating in a medium with nonnegligible losses. The reason is singularities encountered in the course of calculation. The remedy is the addition of a magnetic current density which may be chosen zero after one has reached the last singularity but not before.**

Key Words—**Maxwell's theory, electromagnetic waves, nonsinusoidal waves, sequency theory.**

Index Code—**J3d.**

I. Introduction

ELECTROMAGNETIC wave theory has been based on the concept of infinitely extended periodic sinusoidal waves ever since Maxwell published his theory a century ago. On the practical level this worked very well, but on the theoretical level we always had an indication that something was amiss. There was never a satisfactory concept of the propagation velocity[1] of signals within the framework of Maxwell's theory. The often-mentioned group velocity fails on two accounts, one being that it is almost always larger than the velocity of light in radio transmission through the atmosphere, the other that its derivation implies a transmission rate of information equal to zero.

Beyond the velocity of propagation, we search the literature in vain for a solution of Maxwell's equations for a wave with a beginning and an end, that could represent a signal, propagating in a lossy medium. One might think the reason is the practical difficulty of obtaining solutions, but this is only partly correct. The solutions that we will derive are indeed mathematically complex and they can be made useful only by means of computer plots. However, computers have been with us for 40 years, and for at least half this time they were sufficiently sophisticated and accessible to do the required computations. There was also plenty of incentive to study such solutions. In radar, one would like to know the wave produced by the reflection or scattering of a (sinusoidal) pulse rather than a periodic sinusoidal wave, and in the stealth technology one would like to study the absorption of pulses rather than that of infinitely extended periodic waves. Since the enormous efforts devoted by military scientists to these problems have not yielded satisfactory results, it is clear that something more than mathematical and computational difficulties must be the cause.

A closer study shows that the fault lies with Maxwell's equations rather than with their solutions. In general, there can be no solutions for signals propagating in lossy media. Expressed more scientifically, Maxwell's equations fail for waves with nonnegligible relative frequency bandwidth propagating in a medium with nonnegligible losses.

One way of remedying the failure is by the addition of a magnetic current density to Maxwell's equations. But the remedy is even more surprising than the failure, since it is generally agreed that magnetic currents have not been observed and it is known from the study of magnetic monopoles that a magnetic current density can be eliminated or created by means of a so-called duality transformation. The explanation of both riddles is singularities encountered in the course of calculation. If one chooses the current density zero before reaching the last singularity, one obtains no solution; if one does so after reaching the last singularity, one gets a solution.

II. Modified Maxwell Equations

The usual form of Maxwell's equations in the international system of units is

$$\operatorname{curl} \boldsymbol{H} = \partial \boldsymbol{D}/\partial t + \boldsymbol{g} \tag{1}$$

$$-\operatorname{curl} \boldsymbol{E} = \partial \boldsymbol{B}/\partial t \tag{2}$$

$$\operatorname{div} \boldsymbol{D} = \rho$$

$$\operatorname{div} \boldsymbol{B} = 0$$

$$\boldsymbol{D} = \epsilon \boldsymbol{E}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{3}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field strengths, $\boldsymbol{D}$ and $\boldsymbol{B}$ are the electric and magnetic flux densities, $\boldsymbol{g}$ is the electric current density, ρ the electric charge density, ϵ the permittivity, and μ the permeability.

The question has often been raised whether magnetic currents and charges exist. This matter is usually discussed in the literature under the heading *magnetic monopoles*. Jackson [1] showed that one may indeed write Maxwell's equations

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[1] Discussed in the third paper of this set, "Propagation Velocity of Electromagnetic Signals," this issue, pp. 267–272.

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in the form

$$\text{curl } \boldsymbol{H} = \partial \boldsymbol{D}/\partial t + \boldsymbol{g}_e \tag{4}$$

$$-\text{curl } \boldsymbol{E} = \partial \boldsymbol{B}/\partial t + \boldsymbol{g}_m \tag{5}$$

$$\text{div } \boldsymbol{D} = \rho_e$$

$$\text{div } \boldsymbol{B} = \rho_m$$

$$\boldsymbol{D} = \epsilon \boldsymbol{E}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{6}$$

where $\boldsymbol{g}_e$, $\boldsymbol{g}_m$, ρ_e, and ρ_m stand for electric current density, magnetic current density, electric charge density, and magnetic charge density. One may make a so-called duality transformation that makes $\boldsymbol{g}_m$ and ρ_m zero, which brings one back to Maxwell's equations [1]. Jackson draws the following conclusion.

"The invariance of the equations of electrodynamics under duality transformations shows that it is a matter of convention to speak of a particle possessing an electric charge, but no magnetic charge. The only meaningful question is whether or not *all* particles have the same ratio of magnetic to electric charge. If they do, then we can make a duality transformation, choosing $\cdots$ $\rho_m = 0$, $\boldsymbol{g}_m = 0$. We then have Maxwell's equations as they are usually known" [1].

When studying the propagation of waves with general time variation in a medium with ohmic losses, we find that Maxwell's equations (1)–(3) have no solution, but the modified equations (4)–(6) have solutions, even if one makes the transition $\boldsymbol{g}_m \rightarrow 0$ *in the end*. More precisely, one can make this transition after reaching a certain singularity in the course of calculation, but not before.

Maxwell's equations in the form of (1)–(3) permit two classes of solutions: a) *particular solutions* with sinusoidal time variation and sums of such solutions; and b) solutions with general time variation of the *specialized equations* with $\boldsymbol{g} = 0$ almost everywhere, holding for a loss-free medium. There is no solution in the general case of waves with nonperiodic time variation propagating in a lossy medium.

The reason why this startling failure of Maxwell's equations has not been recognized earlier is that almost all published solutions assume sinusoidal time variation. For a long time there were only two important exceptions: the radiation from a charged particle moving with arbitrary velocity studied by Schwarzschild [2] and Abraham [3], and the radiation of a Hertzian electric dipole with a current of arbitrary time variation discussed in the many editions of the book *Theorie der Elektrizität* (*Theory of Electricity*) by Abraham *et al.* [4] that apparently evolved from the solution for the moving charged particle. Only very recently have these solutions for loss-free media been extended and applied to radio engineering [5]–[7]. Particularly, the stealth technology and anti-stealth-radar created interest in the study of the propagation of (sinusoidal) pulses in lossy or absorbing materials.

III. Planar Wave in Conducting Medium

The modified Maxwell equations (4)–(6) without electric and magnetic charges ρ_e and ρ_m, having constant permittivity ϵ, permeability μ, electric conductivity σ, and *magnetic conductivity* s, assume the following form if the electric and magnetic Ohm's laws

$$\boldsymbol{g}_e = \sigma \boldsymbol{E}$$

$$\boldsymbol{g}_m = s\boldsymbol{H} \tag{7}$$

are used:

$$\text{curl } \boldsymbol{H} = \epsilon \partial \boldsymbol{E}/\partial t + \sigma \boldsymbol{E} \tag{8}$$

$$-\text{curl } \boldsymbol{E} = \mu \partial \boldsymbol{H}/\partial t + s\boldsymbol{H} \tag{9}$$

$$\epsilon \text{ div } \boldsymbol{E} = \mu \text{ div } \boldsymbol{H} = 0. \tag{10}$$

Except for the term $s\boldsymbol{H}$, one obtains the same equations from Maxwell's equations (1)–(3). By choosing $s = 0$ at any stage of the calculation, we may thus check what result Maxwell's equations would have yielded.

Consider a planar transverse electromagnetic (TEM) wave propagating in the direction y. A TEM wave requires

$$E_y = H_y = 0 \tag{11}$$

while a planar wave calls for the following relations:

$$\partial E_x/\partial x = \partial E_x/\partial z = \partial E_z/\partial x = \partial E_z/\partial z = 0 \tag{12}$$

$$\partial H_x/\partial x = \partial H_x/\partial z = \partial H_z/\partial x = \partial H_z/\partial z = 0. \tag{13}$$

Writing the operator curl in Cartesian coordinates and introducing the conditions (11)–(13) brings (8) and (9) into the following form:

$$-\partial H_x/\partial y = \epsilon \partial E_z/\partial t + \sigma E_z \tag{14}$$

$$\partial H_z/\partial y = \epsilon \partial E_x/\partial t + \sigma E_x \tag{15}$$

$$\partial E_x/\partial y = \mu \partial H_z/\partial t + sH_z \tag{16}$$

$$-\partial E_z/\partial y = \mu \partial H_x/\partial t + sH_x. \tag{17}$$

With the substitutions

$$E = E_x = E_z$$

$$H = H_x = -H_z \tag{18}$$

one may rewrite the two pairs of equations (14) and (17) as well as (15) and (16) as one pair:

$$\partial E/\partial y + \mu \partial H/\partial t + sH = 0 \tag{19}$$

$$\partial H/\partial y + \epsilon \partial E/\partial t + \sigma E = 0. \tag{20}$$

We solve this pair of equations first for the electric field strength E. This will show that the introduction of the magnetic conductivity is *sufficient* to obtain a solution, even though we will choose $s = 0$ in the end. In a second paper, we will calculate the magnetic field strength from the electric field strength; this will show that the introduction of s is also *necessary* to obtain a solution.

Differentiation of (19) with respect to y and of (20) with respect to t permits the elimination of the magnetic field strength H. We obtain an equation that contains only the electric field strength:

$$\partial^2 E/\partial y^2 - \mu\epsilon\partial^2 E/\partial t^2 - (\mu\sigma + \epsilon s)\partial E/\partial t - s\sigma E = 0. \quad (21)$$

If E is found from this equation, one may obtain H from either (19) or (20). Equation (20) is readily solved:

$$H(y, t) = -\int (\epsilon\partial E/\partial t + \sigma E)\, dy + H_y(t). \quad (22)$$

The term $H_y(t)$ is an integration constant independent of y. One may also calculate $H(y, t)$ from (19). This is now an inhomogeneous ordinary differential equation of first order with constant coefficients, since $\partial E/\partial y$ is a known function:

$$dH/dt + s\mu^{-1}H = -\mu^{-1}\partial E/\partial y. \quad (23)$$

The homogeneous equation is satisfied by

$$H = Ce^{-st/\mu} \quad (24)$$

and variation of the constant $C = C(t)$ brings the solution of the inhomogeneous equation (23)

$$H(y, t) = e^{-st/\mu}\left[-\mu^{-1}\int (\partial E/\partial y)e^{st/\mu}\, dt + H_t(y)\right] \quad (25)$$

where $H_t(y)$ is an integration constant independent of t.

A partial differential equation like (21) by itself does not define a physical problem. One needs, in addition, boundary and initial conditions. Consider numerous electrodes in the plane $y = 0$. We may use them to apply an electric field strength with the time variation of a step function[2] $S(t)$:

$$E(0, t) = E_0 S(t) = 0, \qquad \text{for } t < 0$$
$$= E_0, \qquad \text{for } t \geqq 0. \quad (26)$$

At the plane $y \to \infty$, we have the further boundary condition

$$E(\infty, t) = \text{finite}. \quad (27)$$

Let E and H be zero for $y > 0$ at the time $t = 0$. We have then the initial conditions

$$E(y, 0) = H(y, 0) = 0. \quad (28)$$

If $E(y, 0)$ and $H(y, 0)$ are zero for all values of $y > 0$, their derivatives with respect to y must be zero too:

$$\partial E(y, 0)/\partial y = \partial H(y, 0)/\partial y = 0. \quad (29)$$

[2] The step function is a particular solution for transients just as a sinusoidal function is a particular solution for the steady state. General periodic steady-state solutions are obtained by means of sums of sinusoidal functions using the Fourier series. Similarly, general transient solutions are obtained by means of sums of time-shifted step functions $A(i)S(t - i\Delta T)$, which for $\Delta T \to dt$ lead to the DuHamel integral. This makes the results obtained for the step function generally applicable for transients that are zero for $t < 0$ and have an arbitrary time variation for $t \geqq 0$. Which transients can be approximated by a sum of step functions is a mathematical problem of the same type as the approximation of periodic functions by a Fourier series. Transient electric and magnetic field strengths in an experimental science can always be approximated. For more details see [8].

Equations (28) and (29) also imply the initial conditions

$$\partial E(y, t)/\partial t = \partial H(y, t)/\partial t = 0 \quad (30)$$

for $y > 0$ and $t = 0$ according to (19) and (20).

We assume that the solution of (21) can be written in the form

$$E(y, t) = E_E(y, t) = E_0[w(y, t) + F(y)] \quad (31)$$

where the notation $E_E(y, t)$ indicates that the electric field strength is excited by the electric step function $E_0S(t)$ of (26); it is also possible to use a magnetic step function $H_0S(t)$ for excitation, which would lead to the electric field strength $E_H(y, t)$.

Insertion of $F(y)$ into (21) yields the equation

$$d^2F/dy^2 - s\sigma F = 0 \quad (32)$$

with the general solution

$$F(y) = A_{00}e^{-y/L} + A_{01}e^{y/L}, \qquad L = (s\sigma)^{-1/2}. \quad (33)$$

The boundary conditions of (26) and (27) require $A_{01} = 0$ and $A_{00} = 1$:

$$F(y) = e^{-y/L}. \quad (34)$$

For the calculation of $w(y, t)$ of (31), we observe that the introduction of the function $F(y)$ transforms the boundary condition of (26) for $E = E_E$ into a homogeneous boundary condition for w

$$E_E(0, t) = E_0 w(0, t) + E_0 = E_0, \qquad \text{for } t \geqq 0 \quad (35)$$

$$w(0, t) = 0, \qquad \text{for } t \geqq 0 \quad (36)$$

while (27) yields

$$w(\infty, t) = \text{finite}. \quad (37)$$

The initial conditions of (28) and (30) yield

$$w(y, 0) + F(y) = 0$$
$$w(y, 0) = -e^{-y/L} \quad (38)$$

$$\partial w(y, t)/\partial t = 0, \qquad \text{for } t = 0, y > 0. \quad (39)$$

Insertion of (31) into (21) yields for $w(y, t)$ the same equation as for $E(y, t)$:

$$\partial^2 w/\partial y^2 - \mu\epsilon\partial^2 w/\partial t^2 - (\mu\sigma + \epsilon s)\partial w/\partial t - s\sigma w = 0. \quad (40)$$

Particular solutions $w_\kappa(y, t)$ are obtained by the separation of variables,

$$w_\kappa(y, t) = \varphi(y)\psi(t) \quad (41)$$

$$\varphi^{-1}\partial^2\varphi/\partial y^2 = \mu\epsilon\psi^{-1}\partial^2\psi/\partial t^2 + (\mu\sigma + \epsilon s)\psi^{-1}\partial\psi/\partial t + s\sigma = -(2\pi\kappa)^2 \quad (42)$$

which yields two ordinary differential equations

$$\partial^2\varphi/\partial y^2 + (2\pi\kappa)^2\varphi = 0 \quad (43)$$

and

$$\partial^2\psi/\partial t^2 + c^2(\mu\sigma + \epsilon s)\partial\psi/\partial t + [(2\pi\kappa c)^2 + s\sigma c^2]\psi = 0, \qquad c^2 = 1/\mu\epsilon \quad (44)$$

with the solutions

$$\varphi(y) = A_{10} \sin 2\pi\kappa y + A_{11} \cos 2\pi\kappa y \quad (45)$$

$$\psi(t) = A_{20} \exp (\gamma_1 t) + A_{21} \exp (\gamma_2 t). \quad (46)$$

The coefficients γ_1 and γ_2 are the roots of the equation

$$\gamma^2 + c^2(\mu\sigma + \epsilon s)\gamma + [(2\pi\kappa)^2 + s\sigma]c^2 = 0 \quad (47)$$

which we write in the following form:

$$\gamma_1 = -a + (a^2 - b^2c^2)^{1/2}, \qquad \text{for } a^2 > b^2c^2 \quad (48)$$

$$\gamma_2 = -a - (a^2 - b^2c^2)^{1/2}$$

$$\gamma_1 = -a + j(b^2c^2 - a^2)^{1/2}, \qquad \text{for } a^2 < b^2c^2$$

$$\gamma_2 = -a - j(b^2c^2 - a^2)^{1/2}$$

$$a = (c^2/2)(\mu\sigma + \epsilon s) = (c/2)(Z\sigma + s/Z) = \sigma/2\epsilon + s/2\mu$$

$$b^2 = (2\pi\kappa)^2 + s\sigma$$

$$c = (\mu\epsilon)^{-1/2}$$

$$Z = (\mu/\epsilon)^{1/2}$$

$$\epsilon = 1/Zc$$

$$\mu = Z/c.$$

Note that Z is used as an abbreviation for $(\mu/\epsilon)^{1/2}$, which is *not* the impedance of a conducting medium as defined in the conventional theory of sinusoidal waves.

The boundary condition (36) requires $A_{11} = 0$ in (45). The particular solution $w_\kappa(y, t)$ thus becomes

$$w_\kappa(y, t) = [A_1 \exp (\gamma_1 t) + A_2 \exp (\gamma_2 t)] \sin 2\pi\kappa y. \quad (49)$$

The general solution $w(y, t)$ is found by making A_1 and A_2 functions of the wavenumber κ, and then integrating over all possible values of κ:

$$w(y, t) = \int_0^\infty [A_1(\kappa) \exp (\gamma_1 t) + A_2(\kappa) \exp (\gamma_2 t)] \sin 2\pi\kappa y \, d\kappa. \quad (50)$$

The time derivative $\partial w/\partial t$ equals

$$\partial w/\partial t = \int_0^\infty [A_1(\kappa)\gamma_1 \exp (\gamma_1 t) + A_2(\kappa)\gamma_2 \exp (\kappa_2 t)] \sin 2\pi\kappa y \, d\kappa. \quad (51)$$

The initial conditions of (38) and (39) demand

$$\int_0^\infty [A_1(\kappa) + A_2(\kappa)] \sin 2\pi\kappa y \, d\kappa = -e^{-y/L} \quad (52)$$

$$\int_0^\infty [A_1(\kappa)\gamma_1 + A_2(\kappa)\gamma_2] \sin 2\pi\kappa y \, d\kappa = 0. \quad (53)$$

These two equations must be solved for the functions $A_1(\kappa)$ and $A_2(\kappa)$. To this end, consider the Fourier sine transform in the following form:

$$g_s(\kappa) = 2 \int_0^\infty f_s(y) \sin 2\pi\kappa y \, dy \quad (54)$$

$$f_s(y) = 2 \int_0^\infty g_s(\kappa) \sin 2\pi\kappa y \, d\kappa.$$

If we identify $2g_s(\kappa)$ first with $A_1(\kappa) + A_2(\kappa)$ and then with $A_1(\kappa)\gamma_1 + A_2(\kappa)\gamma_2$, we obtain from (52) and (53)

$$A_1(\kappa) + A_2(\kappa) = 2g_s(\kappa) = -4 \int_0^\infty e^{-y/L} \sin 2\pi\kappa y \, dy \quad (55)$$

$$A_1(\kappa)\gamma_1 + A_2(\kappa)\gamma_2 = 0. \quad (56)$$

Using the tabulated integral

$$\int_0^\infty e^{uy} \sin 2\pi\kappa y \, dy = \frac{2\pi\kappa}{(2\pi\kappa)^2 + u^2} \quad (57)$$

one obtains from (55) with $L = (s\sigma)^{-1/2}$

$$A_1(\kappa) + A_2(\kappa) = -8\pi\kappa/[s\sigma + (2\pi\kappa)^2] = -8\pi\kappa/b^2 \quad (58)$$

and for the limit $s = 0$

$$A_1(\kappa) + A_2(\kappa) = -2/\pi\kappa. \quad (59)$$

The first reason for the need of a term $s\boldsymbol{H}$ in (9) and $\boldsymbol{g}_m$ in (5) becomes clear now. For $s = 0$, $\sigma \neq 0$, and $L = \infty$, the integral in (55) would not exist. We could have used the standard method of making $\sin 2\pi\kappa y$ integrable by multiplying with a term $e^{-y/L}$ and taking the limit $1/L = 0$. However, the physical meaning of this procedure would have remained unexplained, and the question would be raised why the factor $e^{-y/L}$ should be used and not some other factor or method to obtain integrability. Our approach brings out the physical significance of the integrability. No problem of integrability is encountered if Maxwell's equations are modified by the introduction of a magnetic current density, and the integrability is maintained in the limit of a vanishing magnetic current density.

Equations (56) and (59) are solved for $A_1(\kappa)$ and $A_2(\kappa)$:

$$A_1(\kappa) = -\frac{8\pi\kappa}{b^2}\frac{\gamma_2}{\gamma_2 - \gamma_1} = -\frac{4\pi\kappa}{b^2}\left(1 + \frac{a}{(a^2 - b^2c^2)^{1/2}}\right), \qquad \text{for } a^2 > b^2c^2$$

$$= -\frac{4\pi\kappa}{b^2}\left(1 - \frac{ja}{(b^2c^2 - a^2)^{1/2}}\right), \qquad \text{for } a^2 < b^2c^2$$

$$A_2(\kappa) = -\frac{8\pi\kappa}{b^2}\frac{\gamma_1}{\gamma_1 - \gamma_2} = -\frac{4\pi\kappa}{b^2}\left(1 - \frac{a}{(a^2 - b^2c^2)^{1/2}}\right),$$

$$\text{for } a^2>b^2c^2$$

$$= -\frac{4\pi\kappa}{b^2}\left(1+\frac{ja}{(b^2c^2-a^2)^{1/2}}\right),\quad \text{for } a^2<b^2c^2. \tag{60}$$

Insertion of (48) and (60) into (50) yields

$$\begin{aligned} w(y,\ t) = -e^{-at}\Bigg[\int_0^K \Bigg\{&\left(1+\frac{a}{(a^2-b^2c^2)^{1/2}}\right) \\ &\cdot \exp\,[(a^2-b^2c^2)^{1/2}t] \\ &+\left(1-\frac{a}{(a^2-b^2c^2)^{1/2}}\right) \\ &\cdot \exp\,[-(a^2-b^2c^2)^{1/2}t]\Bigg\}\ \frac{\sin\,2\pi\kappa y}{b^2/4\pi\kappa}\,d\kappa \\ +\int_K^\infty \Bigg\{&\left(1+\frac{ja}{(b^2c^2-a^2)^{1/2}}\right) \\ &\cdot \exp\,[j(b^2c^2-a^2)^{1/2}t] \\ &+\left(1-\frac{ja}{(b^2c^2-a^2)^{1/2}}\right) \\ &\cdot \exp\,[-j(b^2c^2-a^2)^{1/2}t]\Bigg\}\ \frac{\sin\,2\pi\kappa y}{b^2/4\pi\kappa}\,d\kappa\Bigg] \end{aligned} \tag{61}$$

$$K=(2\pi)^{-1}(a^2/c^2-s\sigma)^{1/2}$$

$$b^2=(2\pi\kappa)^2+s\sigma$$

$$a=(c/2)(Z\sigma+s/Z).$$

The imaginary terms in the second integral may be rewritten in real form by means of the formulas

$$e^{jq}+e^{-jq}=2\ \cos\ q$$

$$-j(e^{jq}-e^{-jq})=2\ \sin\ q$$

while the first integral can be simplified with the help of hyperbolic functions:

$$e^q+e^{-q}=2\ \text{ch}\ q$$

$$e^q-e^{-q}=2\ \text{sh}\ q.$$

One obtains

$$\begin{aligned} w(y,\ t) = -\frac{2}{\pi}\,e^{-at}\Bigg\{&\int_0^{2\pi K}\left[\text{ch}\ (a^2-b^2c^2)^{1/2}t + \frac{a\ \text{sh}\ (b^2c^2-a^2)^{1/2}t}{(a^2-b^2c^2)^{1/2}}\right]\frac{\sin\,2\pi\kappa y}{b^2}\,(2\pi\kappa)\ d(2\pi\kappa) \\ &+\int_{2\pi K}^\infty\left[\cos\ (b^2c^2-a^2)^{1/2}t + \frac{a\ \sin\ (b^2c^2-a^2)^{1/2}t}{(b^2c^2-a^2)^{1/2}}\right] \\ &\cdot\frac{\sin\,2\pi\kappa y}{b^2}\,(2\pi\kappa)\ d(2\pi\kappa)\Bigg\}. \end{aligned} \tag{62}$$

To obtain $E_E(y,\ t)$ we still have to add $F(y)$ to $w(y,\ t)$ according to (31). With (34) we get

$$E_E(y,\ t)=E_0[e^{-y/L}+w(y,\ t)]$$

$$L=(s\sigma)^{-1/2}. \tag{63}$$

We now make the transition to $s = 0$. From (48) and (61) we get

$$b=\beta=2\pi\kappa$$

$$a=\alpha=Zc\sigma/2$$

$$2\pi K=\alpha/c=Z\sigma/2. \tag{64}$$

Equations (62) and (63) become

$$\begin{aligned} w(y,\ t) = -\frac{2}{\pi}\,e^{-\alpha t}\Bigg\{&\int_0^{Z\sigma/2}\left[\text{ch}\ (\alpha^2-\beta^2c^2)^{1/2}t + \frac{\alpha\ \text{sh}\ (\alpha^2-\beta^2c^2)^{1/2}t}{(\alpha^2-\beta^2c^2)^{1/2}}\right]\frac{\sin\ \beta y}{\beta}\,d\beta \\ &+\int_{Z\sigma/2}^\infty\left[\cos\ (\beta^2c^2-\alpha^2)^{1/2}t + \frac{\alpha\ \sin\ (\beta^2c^2-\alpha^2)^{1/2}t}{(\beta^2c^2-\alpha^2)^{1/2}}\right]\frac{\sin\ \beta y}{\beta}\,d\beta\Bigg\} \end{aligned} \tag{65}$$

$$E_E(y,\ t)=E_0[1+w(y,\ t)]. \tag{66}$$

In order to get some understanding of the physical content of this equation, we observe that the first integral vanishes when the conductance σ approaches zero. We see that $\alpha = Z\sigma c/2$ also equals zero in this case:

$$\begin{aligned} E_E(y,\ t) &= E_0\left(1-\frac{2}{\pi}\int_0^\infty\frac{\cos\ \beta ct\ \sin\ \beta y}{\beta}\,d\beta\right) \\ &= E_0\left[1-\frac{1}{\pi}\left(\int_0^\infty\frac{\sin\ \beta(y+ct)}{\beta}\,d\beta + \int_0^\infty\frac{\sin\ \beta(y-ct)}{\beta}\,d\beta\right)\right]. \end{aligned} \tag{67}$$

From a table of integrals we find:

$$\int_0^\infty \frac{\sin pq}{q}\, dq = \pi/2, \qquad \text{for } p>0$$
$$= 0, \qquad \text{for } p=0$$
$$= -\pi/2, \qquad \text{for } p<0. \tag{68}$$

Equation (67) thus assumes the following form:

$$E_E(y,\ t) = E_0, \qquad \text{for } y=0,\ t=0$$
$$= 0, \qquad \text{for } y>0,\ ct<y$$
$$= E_0/2, \qquad \text{for } y>0,\ ct=y$$
$$= E_0, \qquad \text{for } y \geqq 0,\ ct>y. \tag{69}$$

This represents a step function with amplitude E_0 propagating with velocity c toward increasing values of y. A look at (21) shows that for $s = 0$ and $\sigma = 0$ it becomes the one-dimensional wave equation which should yield the result of (69) for the boundary condition (26), except for the case $ct = y$ when the wave equation yields the result E_0 rather than $E_0/2$. The discrepancy is due to the fact that the Fourier representation of a function converges to the median $(0 + E_0)/2$, if the function has a discontinuity with two limits 0 and E_0 in a point.

Fig. 1(a) shows $E_E(y,\ t)$ as function of t in the point y, while Fig. 1(b) shows it as function of y at the time t.

For $\sigma \neq 0$ one may rewrite (65) into a more compact normalized form. We make the following substitutions:

$$\eta = \beta c/\alpha = 4\pi\kappa/Z\sigma = (4\pi\kappa/\sigma)(\epsilon/\mu)^{1/2}$$
$$\theta = \alpha t = Z\sigma ct/2 = \sigma t/2\epsilon$$
$$\xi = \alpha y/c = Z\sigma y/2 = (\mu/\epsilon)^{1/2}\sigma y/2 \tag{70}$$

and obtain with

$$\beta y = \eta\xi$$
$$d\beta = \alpha c^{-1} d\eta$$
$$d\beta/\beta = d\eta/\eta$$
$$\eta = 1 \ \text{ for } \beta = Z\sigma/2 \tag{71}$$

the result

$$E_E(\xi,\ \theta) = E_0[1 + w(\xi,\ \theta)] \tag{72}$$

$$w(\xi,\ \theta) = -\frac{2}{\pi} e^{-\theta} \left\{ \int_0^1 \left[\operatorname{ch}\, (1-\eta^2)^{1/2}\theta + \frac{\operatorname{sh}\, (1-\eta^2)^{1/2}\theta}{(1-\eta^2)^{1/2}} \right] \frac{\sin\, \xi\eta}{\eta}\, d\eta + \int_1^\infty \left[\cos\, (\eta^2-1)^{1/2}\theta + \frac{\sin\, (\eta^2-1)^{1/2}\theta}{(\eta^2-1)^{1/2}} \right] \frac{\sin\, \xi\eta}{\eta}\, d\eta \right\}.$$

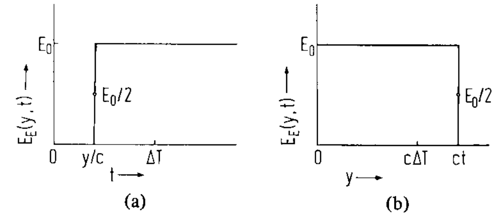

Fig. 1. The field strength $E_E(y,\ t)$ excited by an electric step function in a loss-free medium (a) at the location y as function of the time variable; and (b) at the time t as function of the space variable.

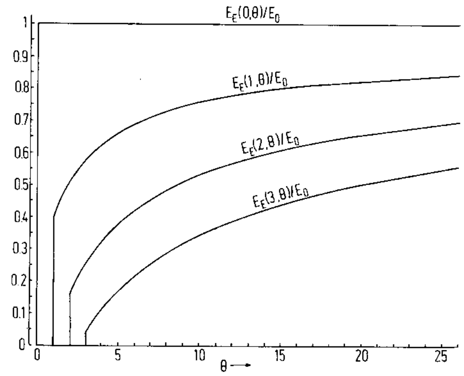

Fig. 2. The function $E_E(\xi,\ \theta)/E_0$ for $\xi = 0, 1, 2, 3$, and θ in the range $0 \leqq \theta \leqq 26$. The nonnormalized variables are $t = \theta/\alpha = (2\epsilon/\sigma)\theta$ and $y = (c/\alpha)\xi = (2/\sigma)(\epsilon/\mu)^{1/2}\xi$. This illustration is based on computer plots by R. Boules, Department of Physics and Applied Mathematics, Faculty of Engineering, University of Alexandria, Alexandria, Egypt.

This equation contains only the normalized space and time variables ξ and θ, since the variable η is eliminated by the integration. All three parameters μ, ϵ, and σ of (8) and (9) have been made part of the normalized variables ξ and θ. A major drawback of (72) is that η becomes infinite for $\sigma = 0$. Hence, the equation is not well suited for the study of the transition to the loss-free medium.

Fig. 2 shows computer plots of $E_E(\xi,\ \theta)$ for the locations $\xi = \alpha y/c = 0, 1, 2, 3$ in the time range $0 \leqq \theta = \alpha t \leqq 26$. The step function $E_E(0,\ \theta)$ at $\xi = 0$ becomes more and more distorted as the distance ξ increases to 1, 2, and 3. However, there is always a jump at $\theta = \xi$; it can be shown analytically that the derivative is infinite at this point for all finite values of ξ. The velocity of propagation of the jump equals $\xi/\theta = (\alpha y/c)/\alpha t$ or $y/t = c$. Assuming an infinite signal-to-noise ratio and detection equipment with infinitely fine resolution, the propagation velocity of the step function thus equals the velocity of light. However, we will clearly need an investigation of finite signal-to-noise ratios and detection equipment with finite resolution.

The plots of Fig. 2 prove that the modification of Maxwell's equations by a magnetic current density is a *satisfactory condition* for obtaining a solution representing a wave excited by an electric step function and propagating in a lossy medium; the solution can be extended to any transient that can be represented by a sum of time-shifted step functions. What is still needed is a proof that the modification by a magnetic current density is also a *necessary condition*. We will show this in a second paper by calculating the magnetic field strength $H_E(\xi, \theta)$ corresponding to the electric field strength $E_E(\xi, \theta)$ of Fig. 2.

IV. Conclusions

Many solutions of Maxwell's equations have been found for infinitely extended periodic sinusoidal waves, and a few solutions were found for nonsinusoidal waves in a loss-free medium, but none has been found for (sinusoidal) pulses or generally transients in lossy media, even though this case is of great practical interest for the reflection or scattering of radar pulses at the ground or water, and the absorption of radar pulses by absorbing materials in the stealth technology. It is shown that a *satisfactory condition*[3] for the existence of solutions for transients in lossy media is the modification of Maxwell's equations by the addition of a magnetic current density. This current density can be chosen zero *after* one reaches a certain singularity in the course of calculation. The result applies to transients with any time variation, as long as they can be approximated by a sum of time-shifted step functions.

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[3]Editor's note: Not necessarily the *only* satisfactory condition.

Summary of Reviewers' Comments and Author's Replies on "Correction of Maxwell's Equations for Signals I"

Overall Comment (by Reviewer)

The title of the paper and the abstract are misleading and the central statements of the paper are invalid. The paper does *not* give a correction to Maxwell's equations nor does it contain any new ideas.

Electromagnetic wave theory has been based on Maxwell's equations and not "on the concept of infinitely extended, periodic sinusoidal waves." Maxwell's equations have been applied with success to problems involving continuous waves as well as to transients and pulses. The other statements in the abstract are also not true. There have been satisfactory treatments of such topics as the propagation velocity of signals in dispersive media. Of course, in material media, Maxwell's equations must be supplemented by constitutive relations which are based on models of the structure of these materials. One may question the dispersion relations derived from these models, but there has been no question about the applicability of Maxwell's equations to macroscopic problems.

Author's Reply

I give references [2]–[7] which are about transients of electromagnetic waves. All of them consider transients in a loss-free medium. The reviewer claims there have been satisfactory treatments of such topics as the propagation velocity of signals in dispersive media, but gives no reference. The catch may be that I consider a medium with *ohmic losses* while the reviewer thinks of a *dispersive* medium, which may not be the same (e.g., Kraus and Carver, *Electromagnetics*, 2nd ed. (New York: McGraw-Hill, 1973), footnote p. 529) say, "The waveguide behaves like a lossless dispersive medium"). Obviously, my paper must sound wrong if the distinction between dispersive medium and (ohmic) lossy medium is not made.

Major Flaw

The main thesis of the paper contains a flaw. As the author admits (in referring to Jackson), there is no need to introduce the concept of a magnetic current, although the introduction of the concept is often useful in solving practical problems with the aid of the principle of duality. In the particular transient problem (involving a unit-step input) dealt with in the paper, the author claims he *must* use magnetic currents to get a solution. This is not true. Maxwell's equations *do* have solutions for signals propagating in lossy media. The difficulty the author encounters does *not* lie with Maxwell's equations but with the *method* the author chooses to solve them. Since the Fourier transform of a step function does not exist, one has two choices: introduce a convergence factor, or use the Laplace transform. The author uses the first choice and claims that this somehow legitimizes the need for a magnetic current. The second choice makes this unnecessary.

Author's Reply

I am misunderstood to claim that a magnetic current density must be added to Maxwell's equations. I only want to claim that Maxwell's equations must be modified, and that the addition of a magnetic current density is a sufficient modification. Dirac's equations and the Klein-Gordon equation are both possible generalizations of the nonrelativistic Schroedinger equation. Similarly, there may be more than one possible generalization of Maxwell's equations.

The reviewer is wrong in making a distinction between introduction of "a convergence factor" and "use of the Laplace transform." There is no way of telling whether the right side of (55) is a Fourier transform with convergence factor or a Laplace transform. The text following (59) explains that my approach 1) shows the *physical significance* of (55) resulting from the imposition of initial conditions, and 2) it

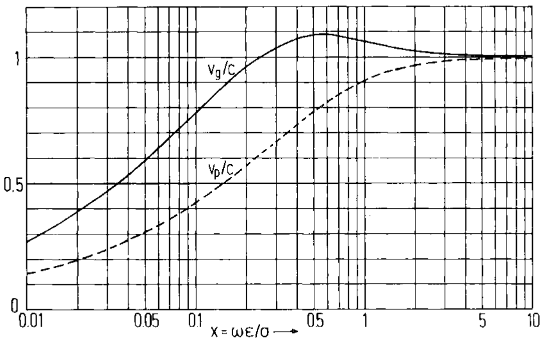

Fig. 2. (Taken from Harmuth, "Propagation Velocity of Electromagnetic Signals," this issue, pp. 267–272.) Normalized phase velocity v_p/c and group velocity v_g/c as function of the normalized frequency $\omega\epsilon/\sigma$ for a conducting medium. For $v_g/c = 1$ at $x = 0.25$: seawater—$f = 225$ MHz; freshwater—$f = 56$ kHz; ionized layers of atmosphere—$f = 50$ Hz; nonionized layers of atmosphere—$f \ll 50$ Hz. Propagation constant:

$$\gamma = [j\omega\mu(j\omega\epsilon + \sigma)]^{1/2} = \alpha + j\beta$$

$$\alpha = \omega\{(\mu\epsilon/2)[(1 + \sigma^2/\omega^2\epsilon^2)^{1/2} - 1]\}^{1/2}$$

$$\beta = \omega\{(\mu\epsilon/2)[(1 + \sigma^2/\omega^2\epsilon^2)^{1/2} + 1]\}^{1/2}$$

$$v_g = \frac{d\omega}{d\beta} = \frac{1}{d\beta/d\omega} = \frac{2\sqrt{2}\{x[x + (x^2+1)^{1/2}](x^2+1)\}^{1/2}}{2[x + (x^2+1)^{1/2}](x^2+1)^{1/2} - 1}\, c$$

$$x = \omega\epsilon/\sigma$$

ϵ = permittivity, μ = permeability, σ = conductivity.

Propagation constant γ may be found in many books, e.g., Kraus and Carver, *Electromagnetics*, 2nd ed. (New York: McGraw-Hill, 1973), p. 404, eq. 9.

explains why this and no other method to obtain convergence is used; simple introduction of a Laplace transform would leave both questions unanswered.

Group Velocity

The author states that group velocity is almost always larger than velocity of light. This does not sound correct.

Author's Reply

This reviewer relies on his belief. I refer to Fig. 2 of the third paper of this set ("Propagation Velocity of Electromagnetic Signals") that shows that the group velocity in the atmosphere is indeed practically always larger than c, whether this sounds correct or not. Enough information is given in Fig. 2 to enable anyone to check my plots, based on the universally accepted propagation constant.

Group Velocity

In the first part of the paper (Section I and the first part of Section II) the author claims that Maxwell's equations have some flaw which results in group velocities greater than the speed of light. Group velocities greater than the speed of light are explainable (see Stratton's book *Electromagnetic Theory* (New York: McGraw-Hill, 1941), sections 5.17 and 5.18). The author should either remove the reference to propagation velocities or explain why the conventional explanation of Stratton is invalid.

Author's Reply

There is nothing wrong with a group velocity larger than c, but it is against the special theory of relativity to interpret velocities larger than c as propagation velocity of information or energy. Maxwell's equations are Lorentz invariant, even though they predate the theory of relativity by 20 years. Stratton's section 5.18 is wrong because it is based on papers by Sommerfeld and Brillouin referenced in the third paper of this set. Sommerfeld and Brillouin obtained their results by adding assumptions about the atomistic structure of matter to Maxwell's theory. This was acceptable in 1914, but today we know that this calls for quantum electrodynamics which did not exist in 1914.

Errors in Stratton

The author also claims that no solutions to Maxwell's equations have appeared for signals of finite bandwidth and in a lossy medium. I refer the author to section 5 of Stratton, and in particular sections 5.9–5.11. The work of Stratton does not necessarily invalidate the author's work. However, he should either drop/modify his claims or point out the errors in Stratton. The author may also want to look at J. Van Bladel's book *Electromagnetic Fields* (New York: McGraw-Hill, 1964), chapter 8.

Author's Reply

The calculation by Stratton in section 5.9 is indeed faulty. He uses particular sinusoidal solutions, as I do, to satisfy Maxwell's equations, and he uses a Fourier transform (p. 297, eq. 64) to satisfy the boundary conditions, as I do, but he does nothing to satisfy the initial conditions for a signal, $E = 0$, $H = 0$ for $t \leqq 0$. His solutions are periodically repeated pulses, not transients or signals. My book [8] treats this point in great detail since the same mistake is made almost universally, but it is difficult to include so many details in these three papers. The closest approach to my results by Stratton is in section 5.13, p. 320 (162); only the electric field strength due to an electric excitation force is calculated, but not the associated magnetic field strength nor the field strengths due to a magnetic excitation force. The equation is based on the assumption of a uniquely defined function and its first derivative in the lines following (146), p. 318, which cannot be met by a signal that must be zero for $t < 0$. (At least one derivative must be two-valued or undetermined at $t = 0$.) This comment is readily understandable for signals represented by a step function $S(t)$ or a linear ramp function $S(t)t$ for small values of t, but the approach fails generally for signals of the form $S(t)t^n$ for small values of t. Anyone trying to derive solutions of Maxwell's equations by means of a straightforward Laplace transformation should carefully read section 5.13 and avoid its error. A Taylor series expansion can never approximate the beginning of a signal, regardless of the number of terms used! The reviewer's expression "signals of infinite bandwidth" is a contradiction in terms. (See "Author's Reply" in "Propagation Velocity of Electromagnetic Signals," this issue, p. 271, end of first paragraph.)

Transition for $s = 0$

The second part of the paper presents the author's derivation which leads to the assertion that the addition of magnetic currents is a sufficient condition for Maxwell's equations

(55), which the author says is not defined for $s = 0$ unless one takes a limiting process. I believe the derivation does contain at least one possible problem. The solution for $F(y)$ in (33) is written as the sum of two exponentials. This solution is valid provided s is not zero. Thus the author's solution only applies for nonzero s, and it is not meaningful to put $s = 0$ into (55).

In summary, the author should: a) consider modifying his original claims based on section 5 of Stratton; and b) show how the solution is modified if $F(y) = Ay + B$ (A, B constants) in (33).

Author's Reply

The reviewer is correct that (55) is of crucial importance. However, I do not see why (33)—or perhaps (34)—prevents one from making the transition $s \to 0$. From (34), one has exp $(-y/L) = \exp(-y\sqrt{s\sigma})$, which becomes 1 for $s = 0$; hence (34) permits the limit $s = 0$. If we take $s = 0$ in (34), we get no solution in (55). The transition $s \to 0$ is only possible after one passes the last singularity, equation (55) for the electric field strength; for the magnetic field strength one must maintain s well into the following paper.

Alternate Calculation

Starting with the author's equation (21) with the magnetic current omitted, we have

$$\partial^2E/\partial y^2 - \mu\epsilon\partial^2E/\partial\tau^2 - \mu\sigma\partial E/\partial\tau = 0.$$

Let $\bar{E}(y, s)$ be the Laplace transform of $E(y, t)$; then the transform of the above equation is

$$\delta^2\bar{E}/\delta y^2 - \mu\epsilon(s^2 + s\sigma/\epsilon)\bar{E} = 0.$$

The integrand has a simple pole at $s = 0$ and branch points at $s = 0$ and $s = -\sigma/\epsilon$. Since $U(s) \to 1$ when $|s| \to \infty$, the Bromwich contour can be closed by a semicircle in the left half-plane for $t < y/c$ and by a semicircle in the right half-plane for $t > y/c$. In the first case, $E(y, t) = 0$. In the second case, the pole contribution gives the first term of (66). The branch cut along the real axis between $s = 0$ and $s = -\sigma/\epsilon = -2\alpha$ contributes a term

$$-\frac{E_0}{\pi}\int_0^{2\alpha} x^{-1} \exp(-xt) \sin[(y/c)\sqrt{x(2\alpha - x)}]\, dx.$$

The substitution $\beta c = \sqrt{x(2\alpha - x)}$ or $x = \alpha \pm \sqrt{\alpha^2 - \beta^2c^2}$ (where the upper sign must be used for $\alpha < x < 2\alpha$ and the lower sign for $0 < x < \alpha$) leads to the author's first integral in (65)—except that the roles of the hyperbolic cosine and sine are interchanged. The branch cut contribution vanishes for zero conductivity. It is not clear how the second integral in (65) arises or why it is needed. The step response is already taken care of in the first term of (66).

By using the shifting theorem and then the initial-value theorem on $E(y, s)$, the step at $t = y/c$ can be shown to be E_0 exp $(-\epsilon)$ in the author's notation. For $\epsilon = 1, 2, 3$ the values of exp $(-\epsilon)$ are 0.3678, 0.1353, and 0.0498, respectively. The values in Fig. 2 show some computational error. On the other hand, the error may be due to the fact that (66) is not correct.

Author's Reply

The reviewer's attempt to calculate my result—done without reference to an accepted mathematical method such as Fourier's method of standing waves used by me—yields a result different from mine. Does this result satisfy the initial conditions $E = 0$, $H = 0$ for $t \leqq 0$ as my result does? It is a common mistake to forget about the initial conditions and thus obtain a result for periodically repeated pulses rather than one pulse. The reviewer claims—without reference or proof—that his result is correct and mine is wrong, but this is, of course, the whole point of publishing my paper.

A strong argument for the correctness of my results are the plots of Fig. 2 of this paper, which were done independently at two different universities to reduce the probability of mistake. To obtain by mistake functions that are zero for $t < y/c$ and then have a jump is quite improbable. To bolster the case of improbability, we have the plots in Fig. 1 of the following paper, and four more independent sets of plots in my book *Propagation of Nonsinusoidal Electromagnetic Waves* [8].

EMC
